

## **XE - A Engineering Mathematics (Compulsory for all XE Candidates)**

### **Section 1: Linear Algebra**

Algebra of real matrices: Determinant, inverse and rank of a matrix; System of linear equations (conditions for unique solution, no solution and infinite number of solutions); Eigenvalues and eigenvectors of matrices; Properties of eigenvalues and eigenvectors of symmetric matrices, diagonalization of matrices; Cayley-Hamilton Theorem.

### **Section 2: Calculus**

**Functions of single variable:** Limit, indeterminate forms and L'Hospital's rule; Continuity and differentiability; Mean value theorems; Maxima and minima; Taylor's theorem; Fundamental theorem and mean value theorem of integral calculus; Evaluation of definite and improper integrals; Applications of definite integrals to evaluate areas and volumes (rotation of a curve about an axis).

**Functions of two variables:** Limit, continuity and partial derivatives; Directional derivative; Total derivative; Maxima, minima and saddle points; Method of Lagrange multipliers; Double integrals and their applications.

**Sequences and series:** Convergence of sequences and series; Tests of convergence of series with non-negative terms (ratio, root and integral tests); Power series; Taylor's series; Fourier Series of functions of period  $2\pi$ .

### **Section 3: Vector Calculus**

Gradient, divergence and curl; Line integrals and Green's theorem.

### **Section 4: Complex variables**

Complex numbers, Argand plane and polar representation of complex numbers; De Moivre's theorem; Analytic functions; Cauchy-Riemann equations.

### **Section 5: Ordinary Differential Equations**

First order equations (linear and nonlinear); Second order linear differential equations with constant coefficients; Cauchy-Euler equation; Second order linear differential equations with variable coefficients; Wronskian; Method of variation of parameters; Eigenvalue problem for second order equations with constant coefficients; Power series solutions for ordinary points.

## **Section 6: Partial Differential Equations**

Classification of second order linear partial differential equations; Method of separation of variables: One dimensional heat equation and two dimensional Laplace equation.

## **Section 7: Probability and Statistics**

Axioms of probability; Conditional probability; Bayes' Theorem; Mean, variance and standard deviation of random variables; Binomial, Poisson and Normal distributions; Correlation and linear regression.

## **Section 8: Numerical Methods**

Solution of systems of linear equations using LU decomposition, Gauss elimination method; Lagrange and Newton's interpolations; Solution of polynomial and transcendental equations by Newton-Raphson method; Numerical integration by trapezoidal rule and Simpson's rule; Numerical solutions of first order differential equations by explicit Euler's method.

### **SECTION 1: Flow and Fluid Properties**

**Fluid Properties:** Density, viscosity, surface tension, relationship between stress and strain-rate for Newtonian fluids.

**Classification of Flows:** Viscous versus inviscid flows, incompressible versus compressible flows, internal versus external flows, steady versus unsteady flows, laminar versus turbulent flows, 1-D, 2-D and 3-D flows, Newtonian versus non-Newtonian fluid flow.

**Hydrostatics:** Buoyancy, manometry, forces on submerged bodies and its stability.

### **SECTION 2: Kinematics of Fluid Motion**

Eulerian and Lagrangian descriptions of fluid motion. Concept of local, convective and material derivatives. Streamline, streakline, pathline and timeline.

### **SECTION 3: Integral Analysis for a Control Volume**

Reynolds Transport Theorem (RTT) for conservation of mass, linear and angular momentum.

### **SECTION 4: Differential Analysis**

Differential equations of mass and momentum for incompressible flows.

Inviscid flows - Euler equations and viscous flows - Navier-Stokes equations.

Concept of fluid rotation, vorticity, stream function and circulation.

Exact solutions of Navier-Stokes equations for Couette flow and Poiseuille flow, thin film flow.

### **SECTION 5: Dimensional Analysis**

Concept of geometric, kinematic and dynamic similarity.

Buckingham Pi theorem and its applications.

Non-dimensional parameters and their physical significance - Reynolds number, Froude number and Mach number.

### **SECTION 6: Internal Flows**

Fully developed pipe flow.

Empirical relations for laminar and turbulent flows: friction factor, Darcy-Weisbach relation and Moody's chart.

Major and minor losses.

## **SECTION 7: Bernoulli's Equation and its Applications, Potential Flows**

**Bernoulli's equation:** Assumptions and applications.

Flow measurements - Venturi meter, Pitot-static tube and orifice meter.

**Elementary potential flows:** Velocity potential function.

Uniform flow, source, sink and vortex, and their superposition for flow past simple geometries.

## **SECTION 8: External Flows**

**Prandtl boundary layer equations:** Concept and assumptions.

**Boundary layer characteristics:** Boundary layer thickness, displacement thickness and momentum thickness.

Qualitative idea of boundary layer separation, streamlined and bluff bodies, and drag and lift forces.

### **Section 1: Basic Concepts**

Continuum and macroscopic approach; thermodynamic systems (closed and open); thermodynamic properties and equilibrium; state of a system, state postulate for simple compressible substances, state diagrams, paths and processes on state diagrams; concepts of heat and work, different modes of work; zeroth law of thermodynamics; concept of temperature.

### **Section 2: First Law of Thermodynamics**

Concept of energy and various forms of energy; internal energy, enthalpy; specific heats; first law applied to elementary processes, closed systems and control volumes, steady and unsteady flow analysis.

### **Section 3: Second Law of Thermodynamics**

Limitations of the first law of thermodynamics, concepts of heat engines and heat pumps/refrigerators, Kelvin-Planck and Clausius statements and their equivalence; reversible and irreversible processes; Carnot cycle and Carnot principles/theorems; thermodynamic temperature scale; Clausius inequality and concept of entropy; microscopic interpretation of entropy, the principle of increase of entropy, T-s diagrams; second law analysis of control volume; availability and irreversibility; third law of thermodynamics.

### **Section 4: Properties of Pure Substances**

Thermodynamic properties of pure substances in solid, liquid and vapor phases; P-v-T behaviour of simple compressible substances, phase rule, thermodynamic property tables and charts, ideal and real gases, ideal gas equation of state and van der Waals equation of state; law of corresponding states, compressibility factor and generalized compressibility chart.

### **Section 5: Thermodynamic Relations**

T-ds relations, Helmholtz and Gibbs functions, Gibbs relations, Maxwell relations, Joule-Thomson coefficient, coefficient of volume expansion, adiabatic and isothermal compressibilities, Clapeyron and Clapeyron-Clausius equations.

## Section 6: Thermodynamic Cycles

Carnot vapor cycle, ideal Rankine cycle, Rankine reheat cycle, air-standard Otto cycle, air-standard Diesel cycle, air-standard Brayton cycle, vapor-compression refrigeration cycle.

## Section 7: Ideal Gas Mixtures

Dalton's and Amagat's laws, properties of ideal gas mixtures, air-water vapor mixtures and simple thermodynamic processes involving them; specific and relative humidities, dew point and wet bulb temperature, adiabatic saturation temperature, psychrometric chart.

