ELECTRICAL ENGINEERING GATE-2021 Forenoon

Section A: GENERAL APTITUDE

1. The people _____________ were at the demonstration were from all sections of society

A. whose

B. which

C. who

D. whom

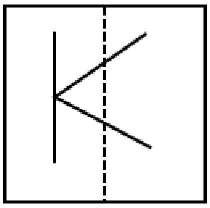
Answer: C

Solution:

We cannot use ‘which’ for people, ‘who’ is the only word which acts as a subject for the verb.

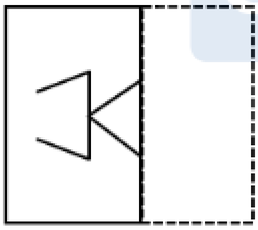
End of the Solution

2.

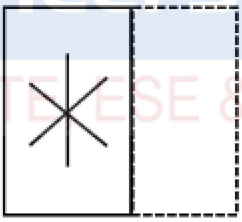


A transparent square sheet shown above is folded along the dotted line. The folded sheet will look like __________

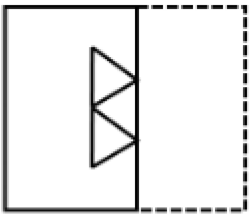
A.



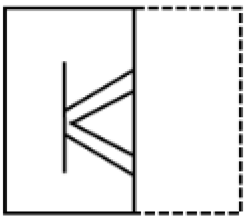
B.



C.



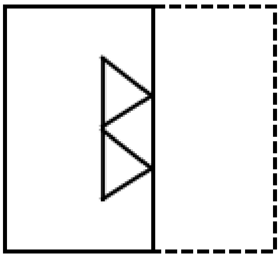
D.



Answer: C

Solution:

Upon rotation, we get the below figure



End of the Solution

3. For a regular polygon having 10 sides, the interior angle between the sides of the polygon, in degrees, is :

A. 396

B. 324

C. 216

D. 144

Answer: D

Solution:

Sum of the interior angle $= (n-2) \times 180 = 1440$

The interior angle between two sides of polygon is $\frac{1440}{10} = 144$.

End of the Solution

4. Which of the following numbers is exactly divisible by $(11^{13}+1)$?

A. $11^{26}+1$

B. $11^{33}+1$

C. $11^{39}-1$

D. $11^{52}-1$

Answer: D

Solution:

Consider expression

$$\frac{x^n - a^n}{x+a}.$$

Now assuming similar expression $11^{52} - 1 = (11^{13})^4 - (1)^4$

$$\frac{(11^{13})^4 - (1)^4}{11^3+1} = \frac{((11)^{13})^2 - (1)^2)((11)^{13})^2 + (1)^2)}{11^3+1}$$

$$[(11^{13}-1)][(11^{13})^2 + (1)^2]$$

Hence, $11^{52} - 1$ is divisible by $11^{13} + 1$.

End of the Solution

5. Oasis is to sand as island is to __________

A. Stone

B. Land

C. Water

D. Mountain

Answer: C

Solution:

Oasis is surrounded by sand as Island is surrounded by water.

End of the Solution

6. The importance of sleep is often overlooked by students when they are preparing for exams. Research has consistently shown that sleep deprivation greatly reduces the ability to recall the material learn. Hence, cutting down on sleep to study longer hours can be counterproductive.

Which of the following statements is the CORRECT inference from the above passage?

A. Sleeping well alone is enough to prepare for an exam. Studying has lesser benefit.

B. Students are efficient and are not wrong in thinking that sleep is a waste of time.

C. If a student is extremely well prepared for an exam, he needs little or no sleep.

D. To do well in an exam, adequate sleep must be part of the preparation.

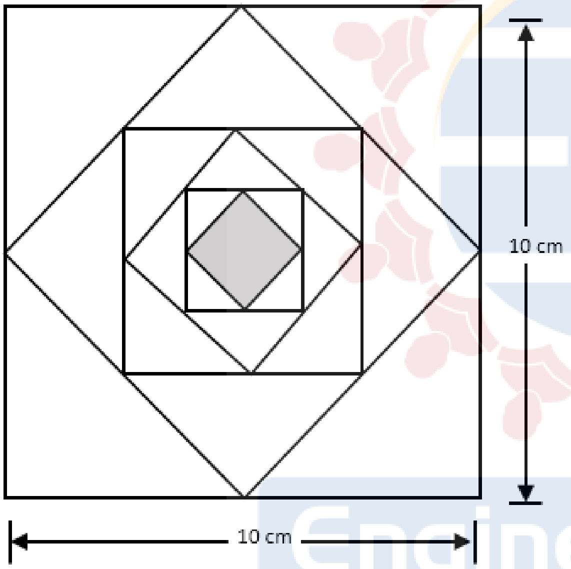
Answer: D

Solution:

First statement is ridiculous, third and fourth are contradictory to what is given in the passage.

End of the Solution

7.



In the figure shown above, each inside square is formed by joining the midpoint of the sides of the next larger square. The area of the smallest square (shaded) as shown, in cm^2 is:

A. 12.50

B. 6.25

C. 3.125

D. 1.5625

Answer: C

Solution:

Side of outer first square $= 10$ cm

Side of outer Second Square $= \frac{10}{\sqrt{2}}$ cm

like wise

Side of outer Third Square $= \left(\frac{10}{(\sqrt{2})^2}\right)$

Hence

Side of smallest square $= \frac{10}{(\sqrt{2})^5}$

Area of smallest square $= \left(\frac{10}{(\sqrt{2})^5}\right)^2 = \frac{100}{2^5} = 3.125$

End of the Solution

8. Let X be a continuous random variable denoting the temperature measured. The range of temperature is [0, 100] degree Celsius and let the probability density function of X be $f(x) = 0.01$ for $0 \le X \le 100$. The mean of X is __________

A. 2.5

B. 5.0

C. 25.0

D. 50.0

Answer: D

Solution:

n is a continuous random variable for continuous random variable

$$\therefore f(x) = \begin{cases} \frac{1}{\beta - \alpha} & \alpha \le x \le \beta \\ 0 & \text{otherwise} \end{cases}$$

From here, $\alpha = 0,\ \beta = 100$

For continuous probability function mean

$$= \frac{\alpha + \beta}{2} = \frac{0 + 100}{2} = 50$$

Alternate Solution:

Given

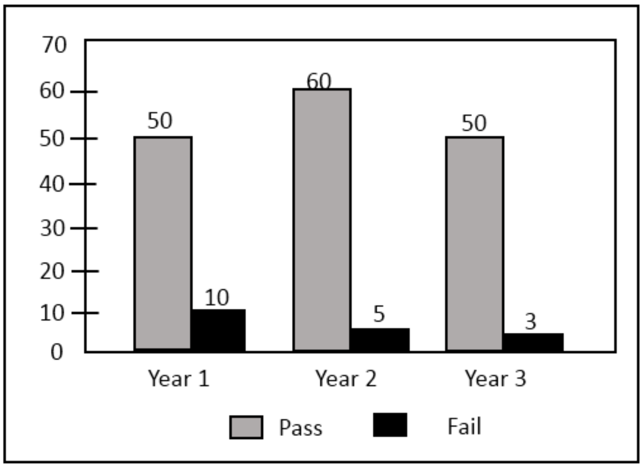
pdf $f(x) = 10^{-2}, 0 < x < 100$

$$E(x) = \int_0^{100} xf(x)dx = \int_0^{100} x10^{-2}dx$$

$$= \frac{1}{100}\frac{x^2}{x}\Big]_0^{100} = \frac{100}{2} = 50$$

End of the Solution

9.



The number of students passing or failing in an exam for a particular subject are presented in the bar chart above. Students who pass the exam cannot appear for the exam again. Students who fail the exam in the first attempt must appear for the exam in the following year, Students always pass the exam in their second attempt.

The number of students who took the exam for the first time in the year 2 and the year 3 respectively, are ____________

A. 65 and 53

B. 60 and 50

C. 55 and 53

D. 55 and 48

Answer: D

Solution:

Total number of student in the year-2

$$= 60 + 5 = 65$$

Students who failed in the year and appeared in year-2 $= 10$

So the students who appeared first time in year-2 $= 65 - 10 = 55$

Similarly, Total number of students in year-3 $= 53$

Students who failed in the year-2 and appeared in year-3 $= 5$

So, the students who appeared first time in year-3 $= 53 - 5 = 48$

End of the Solution

10. Seven cars P, Q, R, S, T, U and V are parked in a row not necessarily in that order. The cars T and U should be parked next to each other. The cars S and V also should be parked next to each other. Whereas P and Q cannot be parked next to each other. Q and S must be parked next to each other. R is parked to the immediate right of V, T is parked to the left of U.

Based on the above statements, the only INCORRECT option given below is:

A. There are two cars parked in between Q and V.

B. Q and R are not parked together.

C. V is the only car parked in between S and R.

D. Car P is parked at the extreme end.

Answer: A

Solution:

∵ S and V must be parked together and Q and S also must be parked together.

These two condition possible parkings are:

Q S V -------

V S Q -------

We can observe that in all cases only one car can be parked in between Q and V which is car S.

Hence option (A) is correct.

End of the Solution

Section B: MATHEMATICS & TECHNICAL

11. Let p and q be real numbers such that $p^2 + q^2 = 1$. The eigenvalue of the matrix $\begin{bmatrix} p & q \\ q & -p \end{bmatrix}$ are

A. 1 and 1

B. 1 and −1

C. j and − j

D. pq and − pq

Answer: B

Solution:

Characteristic equation of A

$$|A_{2\times2} - \lambda I| = (-1)^2\lambda^2 + (-1)^1 \mathrm{Tr}(A)\lambda + |A| = 0$$

$$\lambda^2 - (p - p)\lambda + (-p^2 - q^2) = 0$$

$$\Rightarrow \lambda^2 - 1 = 0$$

$$\Rightarrow \lambda = \pm 1$$

End of the Solution

12. Let $p(z) = z^3 + (1 + j)z^2 + (2 + j)z + 3$, where z a complex number is. Which one of the following is true?

A. Conjugate $\{p(z)\} = p(\text{conjugate}\{z\})$ for all z

B. The sum of the roots of $p(z) = 0$ is a real number

C. The complex roots of the equation $p(z) = 0$ come in conjugate pairs

D. All the roots cannot be real

Answer: D

Solution:

Since sum of the roots is a complex number

⇒ absent one root is complex

So all the roots cannot be real

End of the Solution

13. Let f(x) be a real-valued function such that $f'(x_0) = 0$ for some $x_0 \in (0,1)$, and $f''(x) > 0$ for all $x \in (0,1)$. Then f(x) has

A. no local minimum in (0,1)

B. one local maximum in (0,1)

C. exactly one local minimum in (0,1)

D. two distinct local minima in (0,1)

Answer: C

Solution:

$x_0 \epsilon (0,1)$, where $f'(x) = 0$ is stationary point

$$\text{and } f''(x) > 0$$

$$\forall\, x \in (0,1)$$

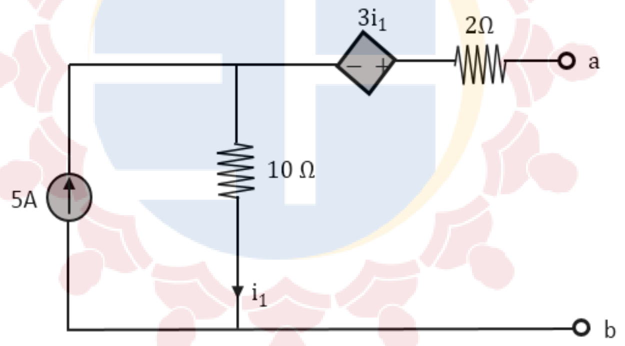
So $f'(x_0) = 0$

and $f''(0) > 0$, where $x_0 \in (0,1)$

Hence, f(x) has exactly one local minima in (0, 1)

End of the Solution

14. For the network shown, the equivalent Thevenin voltage and Thevenin impedance as seen across terminals 'ab' is



A. 10 V in series with 12 Ω

B. 65 V in series with 15 Ω

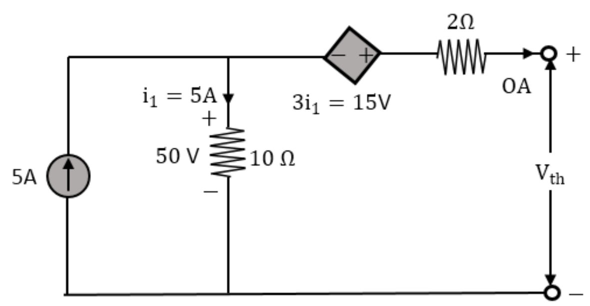
C. 50 V in series with 2 Ω

D. 35 V in series with 2 Ω

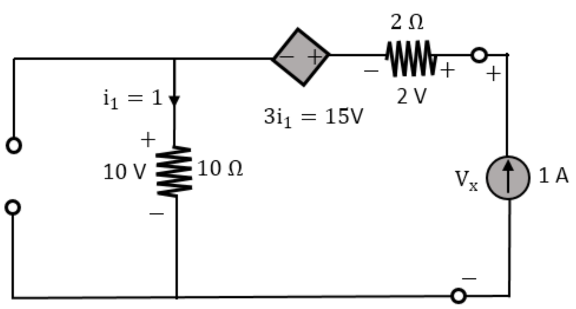
Answer: B

Solution:

Given circuit can be resolved as shown below,



$$V_{TH} = 15 + 50 = 65\text{ V}$$



$$V_x = 2 + 3 + 10 = 15\ V$$

$$R_{TH} = \frac{V_x}{1} = 15\ \Omega$$

End of the Solution

15. Which one of the following vector functions represents a magnetic filed $\vec{B}$? ($\hat{x}, \hat{y}$, and $\hat{z}$ are unit vectors along x-axis, y-axis, and z-axis, respectively)

A. $10x\hat{X} + 20y\hat{Y} - 30z\hat{Z}$

B. $10y\hat{X} + 20x\hat{Y} - 10z\hat{Z}$

C. $10z\hat{X} + 20y\hat{Y} - 30x\hat{Z}$

D. $10x\hat{X} - 30z\hat{Y} - 20y\hat{Z}$

Answer: A

Solution:

If $\vec{B}$ is magnetic flux density then $\vec{\nabla}.\vec{B} = 0$

$$\vec{\nabla}.\vec{B} = \frac{\partial Bx}{\partial x} + \frac{\partial By}{\partial y} + \frac{\partial Bz}{\partial z}$$

$$\frac{\partial}{\partial x}(10x) + \frac{\partial}{\partial y}(20y) + \frac{\partial}{\partial z}(-30z) = \vec{\nabla}.\vec{B}$$

$$\vec{\nabla}.\vec{B} = 10 + 20 - 30 = 0$$

End of the Solution

16. If the input $x(t)$ and output $y(t)$ of a system are related as $y(t) = \max(0, x(t))$, then the system is

A. linear and time-variant

B. linear and time-invariant

C. non-linear and time-variant

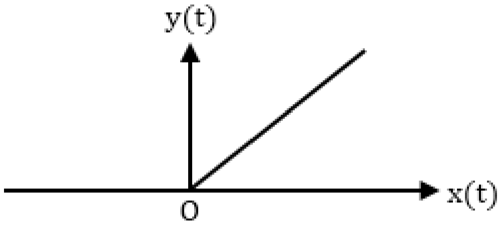
D. non-linear and time-invariant

Answer: D

Solution:

$$y(t) = \max(0, x(t))$$

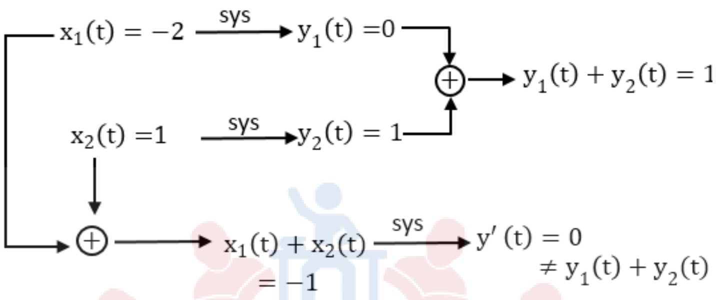
$$= \begin{cases} 0, & x(t) < 0 \\ x(t), & x(t) > 0 \end{cases}$$



Linearity check:

at input $x_1(t) = -2$, output $y_1(t) = 0$

at input $x_2(t) = 1$, output $y_2(t) = 1$



∴ system is non-linear because it violates law of additivity.

Check for time-invariance:

Delayed O/P:

$$y(t - t_0) = \begin{cases} x(t - t_0), & x(t - t_0) > 0 \\ 0 & x(t - t_o) < 0 \end{cases}$$

O/P of system when input is $x(t - t_0) = f(t)$

$$y_1(t) = \begin{cases} f(t), f(t) > 0 \\ 0, f(t) < 0 \end{cases} = \begin{cases} x(t - t_0), x(t - t_0) > 0 \\ 0, \quad x(t - t_0) < 0 \end{cases}$$

Therefore, system is time-invariant.

End of the Solution

17. Two discrete-time linear time-invariant systems with impulse response $h_1[n] = \delta[n-1] + \delta[n+1]$ and $h_2[n] = \delta[n] + \delta[n-1]$ are connected in cascade, where $\delta[n]$ is the Kronecker delta. The impulse response of the cascade system is

A. $\delta[n-2] + \delta[n+1]$

B. $\delta[n-1]\delta[n] + \delta[n+1]\delta[n-1]$

C. $\delta[n-2] + \delta[n-1] + \delta[n] + \delta[n+1]$

D. $\delta[n]\delta[n-1] + \delta[n-2]\delta[n+1]$

Answer: C

Solution:

$h(n)$ =Resultant impulse response

$$= h_1(n) \times h_2(n)$$

By applying z-transform

$$H(z) = H_1(z).H_2(z)$$

$$= (z + z^{-1})(1 + z^{-1})$$

$$= z + z^{-1} + 1 + z^{-2}$$

By applying inverse ZT,

$$h(n) = \delta(n+1) + \delta(n-1) + \delta(n) + \delta(n-2)$$

End of the Solution

18. Consider the table given:

Constructional feature	Machine type	Mitigation
(P) Damper bars	(S) Induction motor	(X) Hunting
(Q) Skewed rotor slots	(T) Transformer	(Y) Magnetic locking
(R) Compensating winding	(U) Synchronous machine	(Z) Armature reaction
	(V) DC machine	

The correct combination that relates the constructional feature, machine type and mitigation is

A. P-V-X, Q-U-Z, R-T-Y

B. P-U-X, Q-S-Y, R-V-Z

C. P-T-Y, Q-V-Z, R-S-X

D. P-U-X, Q-V-Y, R-T-Z

Answer: B

Solution:

End of the Solution

19. Consider a power-system consisting of N number of buses. Buses in this power system are categorized into slack bus, PV buses and PQ buses for load flow study. The number of PQ buses is N_L. The balanced Newton-Raphson method is used to carry out load flow study in polar form. H,S,M, and R are sub-matrices of the Jacobian matrix J as shown below:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = J \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix}, \text{ where } J = \begin{bmatrix} H & S \\ M & R \end{bmatrix}$$

The dimension of the sub-matrix is

A. $N_L \times (N-1)$

B. $(N-1) \times (N-1-N_L)$

C. $N_L \times (N - 1 + N_L)$

D. $(N - 1) \times (N - 1 + N_L)$

Answer: A

Solution:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = J \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix}, \text{Where } J = \begin{bmatrix} H & S \\ M & R \end{bmatrix}$$

For size of M

Row= No. of unknown variables of $Q = N_L$

Column = No. of variable which has $\delta = N_L + (N - 1 - N_L)$

$$= N - 1$$

So, size of $M = N_L \times (N - 1)$

End of the Solution

20. Two generators have cost functions F_1 and F_2. Their incremental-cost characteristics are

$$\frac{dF_1}{dP_1} = 40 + 0.2P_1$$

$$\frac{dF_2}{dP_2} = 32 + 0.4P_2$$

They need to deliver a combined load of 260 MW. Ignoring the network losses, for economic operation, the generation P_1 and P_2 (in MW) are

A. $P_1 = P_2 = 130$

B. $P_1 = 160, P_2 = 100$

C. $P_1 = 140, P_2 = 120$

D. $P_1 = 120, P_2 = 140$

Answer: B

Solution:

$$IC_1 = IC_2$$

$$40 + 0.2P_1 = 32 + 0.4P_2$$

$$0.4P_2 - 0.2P_1 = 8$$

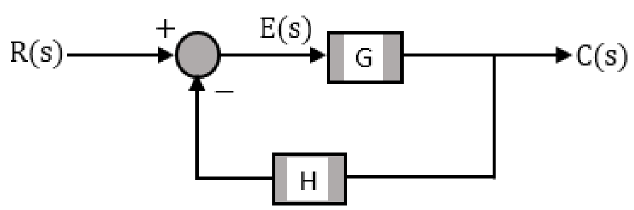
$$P_2 + P_1 = 260$$

Solving equation (i) and (ii),

$$P_1 = 160 \text{ MW}; P_2 = 100 \text{ MW}$$

End of the Solution

21. For the closed-loop system shown, the transfer function $\frac{E(s)}{R(s)}$ is



A. $\frac{G}{1+GH}$

B. $\frac{GH}{1+GH}$

C. $\frac{1}{1+GH}$ D. $\frac{1}{1+G}$

Answer: C

Solution:

$$\frac{E(s)}{R(s)} = \frac{R(s) - H \times C(s)}{R(s)} = 1 - H \times \frac{C(s)}{R(s)}$$

$$= 1 - \frac{HG}{1+GH} = \frac{1+GH-GH}{1+GH}$$

$$= \frac{1}{1+GH}$$

End of the Solution

22. Inductance is measured by

A. Schering bridge B. Maxwell bridge

C. Kelvin bridge D. Wien bridge

Answer: B

Solution:

End of the Solution

23. Suppose the circle $x^2 + y^2 = 1$ and $(x-1)^2 + (y-1)^2 = r^2$ intersect each other orthogonally at the point (u, v). Then $u + v =$ _________.

Answer: 1 to 1

Solution:

If two curves cut orthogonally then product of slopes $= -1$

$$x^2 + y^2 = 1$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$(M_1)_{u,v} = \frac{dy}{dx} = \frac{-x}{y} = \frac{-u}{v}$$

$$(x-1)^2 + (y-1)^2 = r^2$$

$$(M_2)_{u,v} = \left(\frac{dy}{dx}\right) = \left(\frac{-2(x-1)}{2(y-1)}\right)_{(u.v)} = \frac{1-u}{v-1}$$

$$M_1 M_2 = -1$$

$$\frac{-u}{v} \times \frac{1-u}{v-1} = -1$$

$$-u + u^2 = -v^2 + v$$

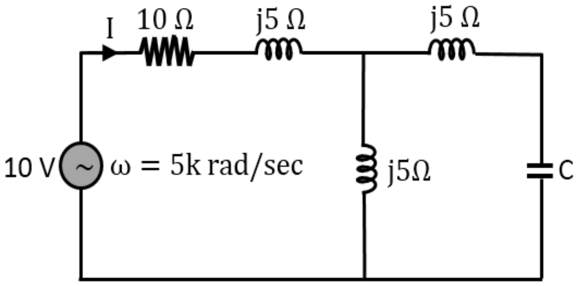
$$u^2 + v^2 = v + u$$

$$x^2 + y^2 = 1$$

$$u + v = 1$$

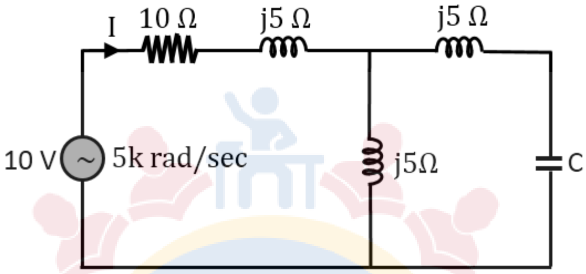
End of the Solution

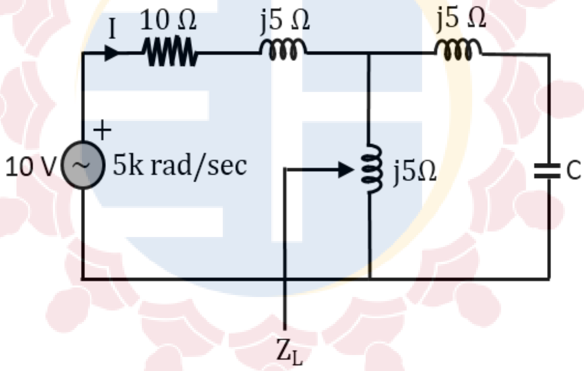
24. In the given circuit, the value of capacitor C that makes current I = 0 is _______μF.



Answer: 20.00 to 20.00

Solution:





$$Z_L = (j5)||(j5 - jX_c)$$

$$\frac{(j5)(j5 - jX_c)}{j5 + j5 - jX_c} = \infty$$

$$j5 + j5 - jX_c = 0$$

$$\Rightarrow jX_c = j10$$

$$\Rightarrow X_c = 10\ \Omega$$

$$X_c = \frac{1}{\omega C} = 10$$

$$\Rightarrow C = \frac{1}{10 \times 5 \times 10^3}$$

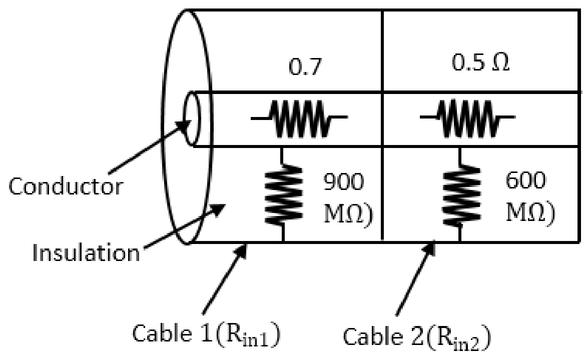
$$C = \frac{1}{5 \times 10^4} \times \frac{10^2}{10^2} = \frac{10^2}{5 \times 10^6} 20\ \mu F$$

End of the Solution

25. Two single-core power cables have total conduction resistances of 0.7Ω and 0.5Ω respectively, and their insulation resistance (between core and sheath) are 600 MΩ and 900 MΩ, respectively. When the two cables are joined in series, the ratio of insulation resistance to conductor resistance is _______$\times 10^6$.

Answer: 300 to 300

Solution:



$$(R_{eq})_{conductor} = 0.7 + 0.5 = 1.2\Omega$$

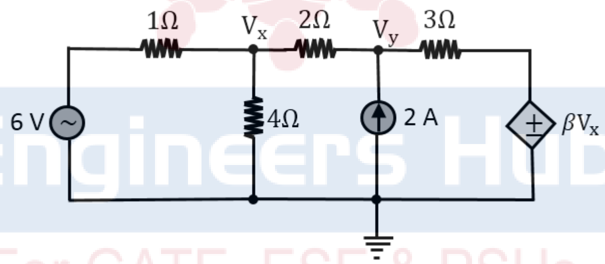
$$(R_{eq})_{in} = \frac{1}{(R_{eq})_{in}} = \frac{1}{(R_{in})_1} + \frac{1}{(R_{in})_2} = \frac{1}{900 \times 10^6} + \frac{1}{600 \times 10^6}$$

$$(R_{eq})_{in} = 360 \times 10^6 \Omega$$

$$\frac{(R_{eq})_{in}}{(R_{eq})_{conductor}} = \frac{360 \times 10^6}{1.2} = 300 \times 10^6.$$

End of the Solution

26. In the given circuit, for voltage V_y to be zero, the value of β should be _______. (Round off to 2 decimal places).



Answer: -3.30 to -3.20

Solution:

$$\frac{V_x - 6}{1} + \frac{V_x}{4} + \frac{V_x - V_y}{2} = 0$$

$$4V_x - 24 + V_x + 2V_x - 2V_y = 0$$

$$7V_x - 2V_y = 24$$

$$\text{IF } V_y = 0$$

$$\Rightarrow 7V_x = 24$$

$$\Rightarrow V_x = \frac{24}{7}$$

$$\frac{V_y - V_x}{2} + \frac{V_y - \beta V_x}{3} = 2$$

$$3V_y - 3V_x + 2V_Y - 2\,\beta V_x = 12$$

$$5V_y - (3 + 2\beta)V_x = 12$$

$$V_y = 0$$

$$-(3 + 2\beta)\frac{24}{7} = 12$$

$$(3 + 2\beta) = \frac{-7}{2}$$

$$2\beta = \frac{-7}{2} - 3 = \frac{-7 - 6}{2} = \frac{-13}{2}$$

$$\beta = \frac{-13}{4} = -3.25$$

End of the Solution

27. A 1 μC point charge is held at the origin of a Cartesian coordinate system. If a second point charge of 10 μC is moved from (0,10,0) to (5,5,5) and subsequently to (5,0,0), then the total work done is ______ mJ. (Rounded off to 2 decimal places).

Take $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ in SI units. All coordinates are in meters.

Answer: 8.90 to 9.10

Solution:

$$E^{\rho} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{a}_r = \frac{10^{-6}(9 \times 10^9)}{r^2}\hat{a}_r = \frac{9 \times 10^3}{r^2}\hat{a}_r$$

In this case work done is independent of type of path but depends an initial and final point.

x	y	z		r	θ	ϕ	
(0,	10,	0)	→	(10,	90°,	90°)	Initial point
x	y	z		r	θ	ϕ	
(5	5,	5)	→	$(5\sqrt{3},$	54.73°, 4	45°)	Intermediate point
x	y	z		r	θ	ϕ	
(5	0,	0)	→	(5,	90°,	0°)	Final point

Work done (by external source) in moving Q-charge (10 μC)) in the presence of electric filed $\vec{E}$ is

$$W = -Q \int_{\text{initial point}}^{\text{final point}} \vec{E}.\,d\vec{l} = -(10 \times 10^{-6}) \int_{r=10}^{r=5} \frac{9 \times 10^3}{r^2}\,\hat{a}_r.\,dr\hat{a}_r$$

$$W = -10 \times 10^{-6}(9 \times 10^3)\left(-\frac{1}{r}\right)_{r=10}^{5}$$

$$= 9 \times 10^{-2}\left[\frac{1}{5} - \frac{1}{10}\right] = 9 \times 10^{-2}\left[\frac{10-5}{50}\right] = 9 \times 10^{-2}(10^{-1})$$

$$W = 9\text{mj}$$

End of the Solution

28. The power input to a 500 V, 50 Hz. 6-pole, 3-phase induction motor running at 975 RPM is 40 kW. The total stator losses are 1 kW. If the total friction and windage losses are 2.025 kW, then the efficiency is ________%.

Answer: 89.50 to 90.50

Solution:

$$P_{i/p} = 40\text{ kW},$$

Stator loss 1kW, F and W = 2.025 kW

$\text{Stator}\frac{0}{p} = 40 - 1 = 39\text{ kW}$

$$\text{Slip} = \frac{1000 - 975}{1000} = 0.025$$

$\text{Rotor}\frac{0}{p} = \text{Rotor}\frac{i}{p} \times (1 - s)$

$$= 39(1 - 0.025) = 38.025\text{ kW}$$

$$\text{Motor}\frac{0}{p} = 38.025 - 2.025 = 36\text{ kW}$$

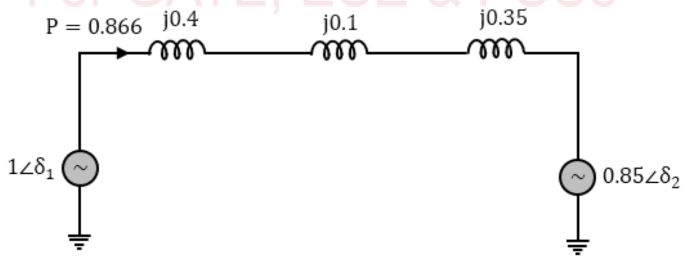
End of the Solution

29. An alternator with internal voltage of $1\angle\delta_1$ p.u and synchronous reactance of 0.4 p.u is connected by a transmission line of reactance 0.1 p.u to a synchronous motor having synchronous reactance 0.35 p.u and internal voltage of $0.85\angle\delta_2$ p.u. If the real power supplied by the alternator is 0.866 p.u, then $(\delta_1 - \delta_2)$ is _____ degrees (Round off to 2 decimal places.)

(Machines are of non-salient type, Neglect resistances)

Answer: 59.00 to 61.00

Solution:



Given real power in (p.u), P = 0.866

$$P = \frac{EV\sin(\delta_1 - \delta_2)}{X_{eq}} = \left|\frac{1 \times 0.85}{0.4 + 0.1 + 0.35}\right|\sin(\delta_1 - \delta_2)$$

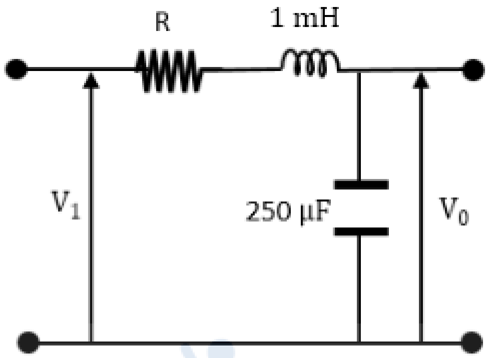
$$(\delta_1 - \delta_2) = 60°$$

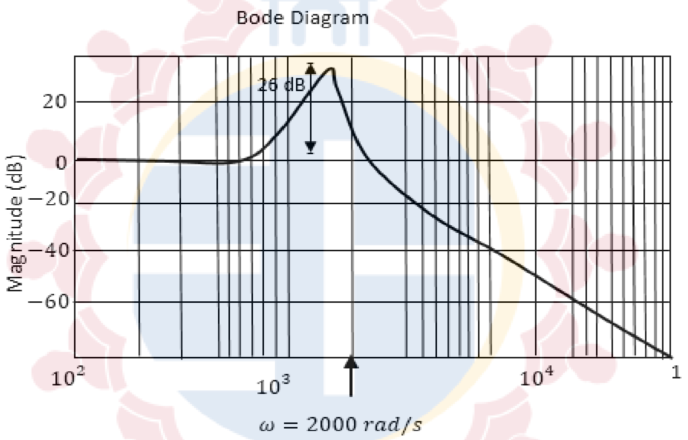
End of the Solution

30. The Bode magnitude plot for the transfer function $\frac{V_o(s)}{V_i(s)}$ of the circuit is as shown. The value of R is ________Ω (Round off to 2 decimal places.).

Answer: 0.09 to 0.11

Solution:





From response plot

$$M_r = 26 \text{ dB} = 20$$

$$\therefore \frac{1}{2\xi\sqrt{1-\xi^2}} = 20$$

$$\therefore \xi = 0.025$$

From electrical network

$$X = \frac{R}{2}\sqrt{\frac{C}{L}} = 0.025$$

$$\therefore R = 0.10\Omega$$

End of the Solution

31. A signal generator having a source resistance of 50 Ω is set to generate a 1 kHz sinewave. Open circuit terminal voltage is 10 V peak-to-peak. Connecting a capacitor across the terminal reduces the voltage to 8 V peak-to-peak. The value of this capacitor is __________μF. (Round off to 2 decimal places.)

Answer: 2.30 to 2.50

Solution:

$$\frac{V_0(j\omega)}{V_i(j\omega)} = \frac{1}{1 + j\omega RC}$$

$$\frac{8}{10}\angle\theta = \frac{1}{1 + j\omega RC}$$

$$1 + j\omega RC = 1.25\angle\theta$$

$$\omega RC = \sqrt{1.25^2 - 1^2}$$

$$\omega RC = 0.75$$

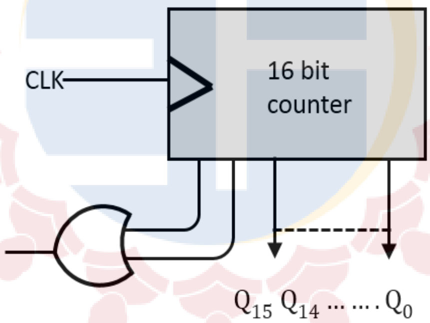
$$C = \frac{0.75}{2000\pi \times 50} = 2.38\ \mu F$$

32. A 16-bit synchronous binary up-counter is clocked with a frequency $f_{CLK.}$ The two most significant bits are OR-ed together to form an output Y. Measurements show that Y is periodic, and the duration for which Y remains high in each period is 24 ms. The clock frequency f_{CLK} is _______ MH_z.

(Round off to 2 decimal places)

Answer: 2.00 to 2.10

Solution:



$$y = 1 \text{ for } 24 \text{ m sec}$$

		Q_{15}	Q_{14}	.....	Q_0
Y=1	Starts at	0100	0000	0000	0000
Y=1	Ends at	1111	1111	1111	1111

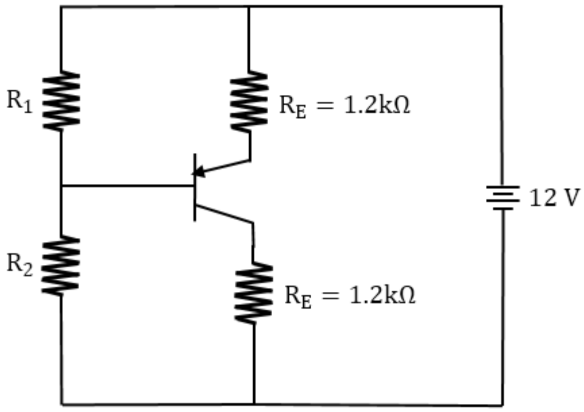
Total number of states for which $y = 1$ is $2^{16} - 2^{14}$

Time is $(2^{16} - 2^{14})T = 24$ msec

$$f = \frac{1}{T} = 2.05\ \text{MHz}$$

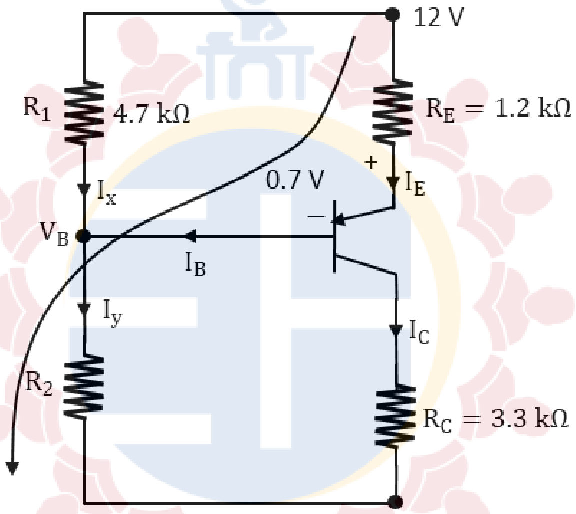
End of the Solution

33. In the BJT circuit shown, beta of the PNP transistor is 100. Assume $V_{BE} = 0.7V$. The voltage across R_c will be 5 V when R_2 is _____kΩ (Round off to 2 decimal places).



Answer: 16.70 to 17.70

Solution:



$$I_C = \frac{5V}{3.3K} = 1.515 \text{ mA}$$

$$I_E = 1.53 \text{ mA}$$

$$I_B = 0.0151 \text{ mA}$$

$$-12 + 1.2k \times 1.53\text{ m} + 0.7 + V_B = 0$$

$$V_B = 9.464 \text{ V}$$

$$I_x = \frac{12 - V_B}{4.7k} = \frac{12 - 9.464}{4.7k} = 0.539 \text{ mA}$$

$$I_x + I_B = I_y$$

$$\Rightarrow I_y = 0.5396 + 0.0151$$

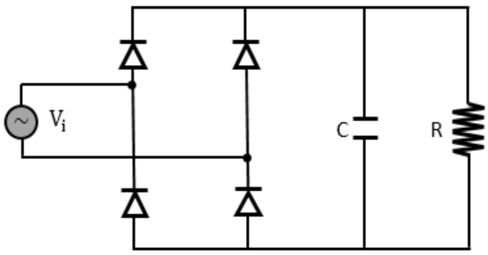
$$I_y = 0.5546 \text{ mA}$$

$$V_B = 0.5546\text{ m} \times R_2 = 9.464$$

$$R_2 = 17.06 \text{ k}\Omega$$

End of the Solution

34. In the circuit shown, the input V_i is a sinusoidal AC voltage having an RMS value of 230 ±20%. The worst-case peak-inverse voltage seen across any diode is ______V. (Round off to 2 decimal places.)



Answer: 389 to 391

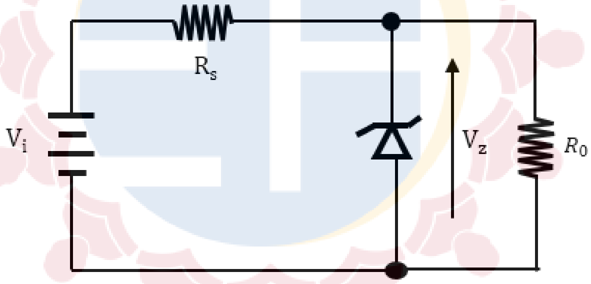
Solution:

$(V_D)_{peak}$ for worst case $= (230 + 20\%) \times \sqrt{2}$

$$= \left(230 + 230 \times \frac{20}{100}\right) \times \sqrt{2} = 390.32\text{ V}$$

End of the Solution

35. In the circuit shown, a 5 V Zener diode is used to regulate the voltage across load R_o. The input is an unregulated DC voltage with a minimum value of 6 V and a maximum value of 8 V. The value of R_s is 6Ω. The Zener diode has a maximum rated power dissipation of 2.5 W. Assuming the Zener diode to be ideal, the minimum value of R_o is ________Ω.



Answer: 29.00 to 31.00

Solution:

To calculate $R_{0,min}$, we must find $I_{L\,max}$

$$I_{s\,min} = I_z\,min + I_{L\,max}$$

$$\frac{V_{i\,min} - V_z}{R_s} = I_z\,min + I_{L\,max}$$

For ideal Zener diode, $I_{z\,min} = 0$

$$\frac{V_{imin} - V_z}{R_s} = I_{L\,max}$$

$$\frac{6-5}{6} = I_{L\,max}$$

$$I_{L\,max} = \frac{1}{6}\text{A}$$

$$R_{0\,min} = \frac{V_Z}{I_{L\,max}} = \frac{5}{1/6} = 30\Omega$$

End of the Solution

36. In the open interval (0,1), the polynomial $p(x) = x^4 - 4x^3 + 2$ has

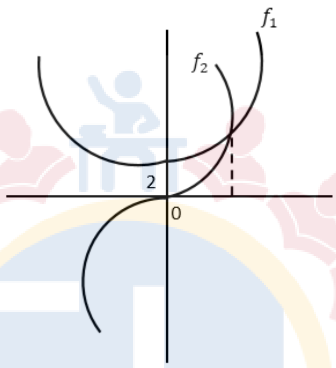
A. two real roots
B. one real root
C. three real roots
D. no real roots

Answer: B

Solution:

$$x^4 + 2 = 4x^3$$
$$f_1(x) = x^4 + 2$$
$$f_2(x) = 4x^3$$

It is clear that point of intersection of these graphs is solution (or) root of $\rho(x) = 0$



According to intermediate value theorem

P(0) and P(1) are having opposite signs

$\therefore$ a root of $\rho(x) = 0$ in (0,1)

and also from graph, there is only one point of intersection

Hence exactly one real root exists in (0,1)

End of the Solution

37. Suppose the probability that a coin toss shows "head" is p, where $0 < p < 1$. The coin is tossed repeatedly until the first "head" appears. The expected number of tosses required is

A. $p/(1-p)$
B. $(1-p)/p$
C. $1/p$
D. $1/p^2$

Answer: C

Solution:

$P(H) = p$, let x = Number of tosses.

x	1	2	3	4	5…
P(x)	p	$(1-p)p$	$(1-p)^2p$		

$$E(x) = \sum x_i p(x) = p + 2(1-p)p + 3(p)(1-p)^2 + \cdots$$
$$= p[1 + 2(1-p) + 3(1-p)^2 + \cdots]$$

$$= p[1-(1-p)]^{-2} = \frac{1}{p}$$

End of the Solution

38. Let $(-1-j), (3-j), (3+j)$ and $(-1+j)$ be the vertices of a rectangle C in the complex plane. Assuming that C is traversed in counter-clockwise direction, the value of the contour integral $\oint_C \frac{dz}{z^2(z-4)}$ is

A. $j\pi/2$

B. 0

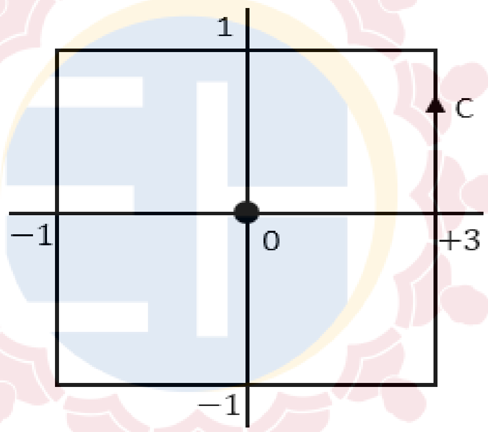
C. $-j\pi/8$

D. $j\pi/16$

Answer: C

Solution:

$$\oint_C \frac{dz}{z^2(z-4)}$$



Singularities are given by $z^2(z-4) = 0$

$$\Rightarrow z = 0,4$$

$z = 0$ is pole of order $m = 2$ lies inside contour ‘c’

$z = 4$ is pole of order , $m = 1$ lies outside ‘c’

$$Res_0 = \frac{1}{(2-1)!} \underset{z\to 0}{lt} \frac{d^{2-1}}{dz^{2-1}}\left[(z-0)^2 \frac{1}{z^2(z+4)}\right]$$

$$= \frac{-1}{(0.4)^2} = \frac{-1}{16}$$

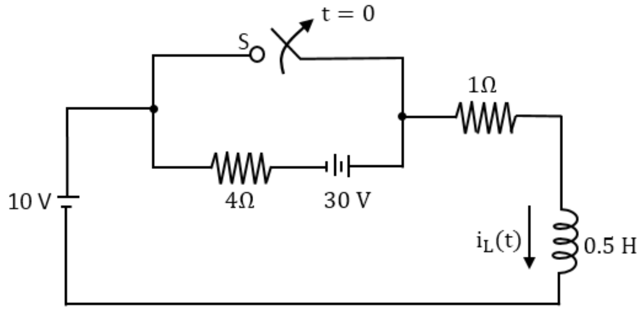
By CRT

$$\oint_C f(z)dz = 2\pi i Res_0 = 2\pi i\left[\frac{-1}{16}\right]$$

$$= \frac{-\pi i}{8}$$

End of the Solution

39. In the circuit, switch ‘S’ in the closed position for a very long time. If the switch is opened at time $t = 0$, then $i_L(t)$ in amperes, for $t \geq 0$ is



A. $8\,e^{-10t}$

B. 10

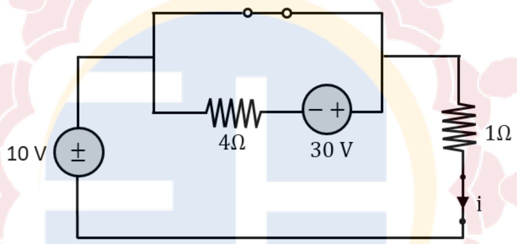
C. $8 + 2e^{-10t}$

D. $10(1 - e^{-2t})$

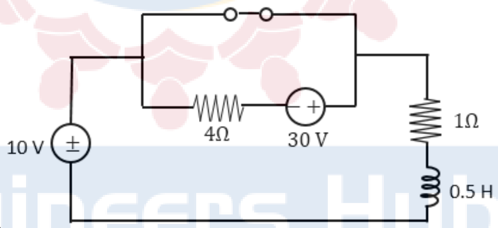
Answer: C

Solution:

At $t = 0^-$

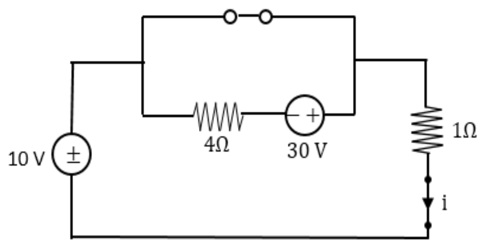


$$i_L(0^-) = \frac{10}{1} = 10\text{ A}$$



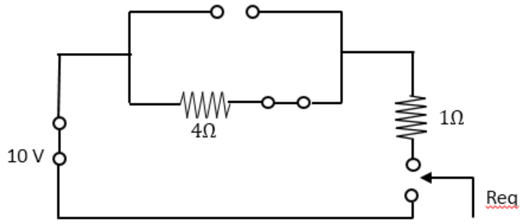
For $t > 0$

At $t = \infty$



$$i(\infty) = \frac{40}{5} = 8\text{A}$$

R_{eq}:



$$R_{eq} = 5\ \Omega$$

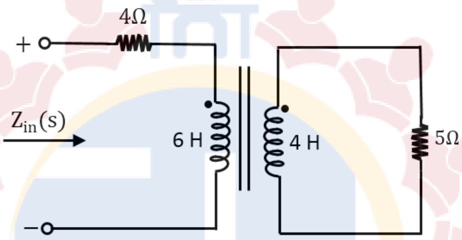
$$\tau = \frac{L}{R_{eq}} = \frac{0.5}{5} = 0.1\text{ Sec}$$

$$i(t) = 8 + [10 - 8]e^{-t/0.1}$$

$$= 8 + 2e^{-10t}\,A$$

End of the Solution

40. The input impedance, $Z_{in}(s)$, for the network shown is



A. $\frac{23s^2+46s+20}{4s+5}$

B. $6s + 4$

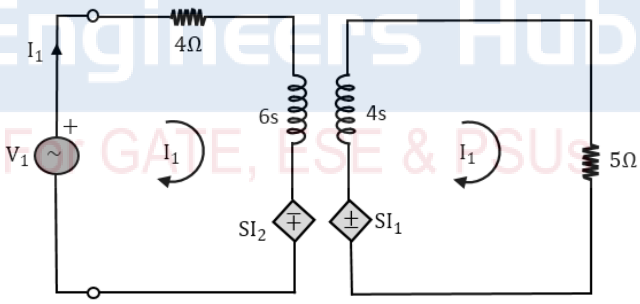
C. $7s + 4$

D. $\frac{25s^2+46s+20}{4s+5}$

Answer: A

Solution:

Circuit in s-domain



$$-SI_1 + (4S + 5)I_2 = 0$$

$$\Rightarrow I_2 = \frac{S}{4S + 5} I_1$$

$$V_1 = (4 + 6S)I_1 - \frac{S^2}{4S + 5}\, I_1$$

$$\frac{V_1}{I_1} = \frac{(4 + 6S)(4S + 5) - S^2}{4S + 5}$$

$$= \frac{24S^2 + 30S + 16S + 20 - S^2}{4S + 5}$$

$$Z_{in} = \frac{23S^2 + 46S + 20}{4S + 5}$$

End of the Solution

41. The causal signal with z-transform $z^2(z - a)^{-2}$ is (u[n] is the unit step signal)

A. $a^{2n}u[n]$

B. $(n + 1)a^n u[n]$

C. $n^{-1}a^n u[n]$

D. $n^2 a^n u[n]$

Answer: B

Solution:

As we know,

$$n.a^n u(n) \rightleftharpoons \frac{az}{(z - a)^2}$$

$$\frac{1}{a}.n.a^n\, u(n) \rightleftharpoons \frac{z}{(z - a)^2}$$

$$f(n) = n.a^{n-1}\, u(n) \rightleftharpoons \frac{z}{(z - a)^2} = F(z)$$

Time-shifting properly,

$$f(n) \rightleftharpoons F(z)$$

$$f(n + 1) \rightleftharpoons z.F(z)$$

$$x(n) = (n + 1)a^n u(n + 1) \rightleftharpoons \frac{z^2}{(z - a)^2} = X(z)$$

Thus, $x(n) = (n + 1)a^n u(n + 1) = (n + 1)a^n.u(n)$

$$[\because (n + 1)u(n + 1) = (n + 1)u(n)]$$

End of the Solution

42. Let f(t) be an even function i.e. $f(-t) = f(t)$ for all t. Let the Fourier transform of f(t) be defined as $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$. Suppose $\frac{dF(\omega)}{d\omega} = -\omega F(\omega)$, and $F(0) = 1$. Then

A. $f(0) < 1$

B. $f(0) > 1$

C. $f(0) = 1$

D. $f(0) = 0$

Answer: A

Solution:

$$f(t) \rightleftharpoons F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

The following information are given about $f(t) \rightleftharpoons F(\omega)$

(i) $f(t) = f(-t)$

(ii) $F(\omega)|_{\omega=0} = 1$

(iii) $\frac{dF(\omega)}{d\omega} = -\omega F(\omega)$

From (iii). $\frac{dF(\omega)}{d\omega} + \omega F(\omega) = 0$

By solving the above linear differential equations. (by mathematics)

$$\text{In } F(\omega) = -\frac{\omega^2}{2}$$

$$\Rightarrow F(\omega) = K.e^{-\omega^2/2}$$

Put $\omega = 0$,

$$F(0) = K$$

$$\Rightarrow 1 = K(\text{from info. (ii)}$$

From (iv), $F(\omega) = e^{-\omega^2/2}$

As we know, $e^{-at^2}, a > 0 \rightleftharpoons \sqrt{\frac{\pi}{a}} e^{-\omega^2/4a}$

At $a = \frac{1}{2}$, $e^{-t^2/2} \rightleftharpoons \sqrt{\frac{\pi}{1/2}}.e^{-\omega^2/2}$

$$f(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2}/2 \rightleftharpoons e^{-\frac{\omega^2}{2}} = F(\omega)$$

Thus, $f(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$

At $t = 0, f(0) = \frac{1}{\sqrt{2\pi}} < 1$

End of the Solution

43. In a single-phase transformer, the total iron loss is 2500 W at nominal voltage of 440 V and frequency 50 Hz. The total iron loss is 850 W at 220 V and 25 Hz. Then, at nominal voltage and frequency, the hysteresis loss and eddy current loss respectively are

A. 1600 W and 900 W

B. 900 W and 1600 W

C. 250 W and 600 W

D. 600 W and 250 W

Answer: B

Solution:

$$W_{i1} = 2500 \text{ W at } 440 \text{ V}, 50 \text{ Hz}$$

$$W_{i2} = 850 \text{ W at } 220 \text{ V}, 25 \text{ Hz}$$

$$W_{i3} = R_{e3} + P_{h3} = \text{at } 440 \text{ V}, 50 \text{ Hz}$$

$R_{e3} = ?, P_{h3,} = ?, \frac{v}{f} = \text{constant}$

$$\left\{\frac{400}{50} = \frac{220}{25} = \text{Constant}\right\}$$

$$\Rightarrow 2500 = Af + Bf^2$$

$$\text{or}, \frac{2500}{f} = A + Bf$$

$$\text{or, } \frac{2500}{50} = A + B(50) - Eq(i)$$

$$\text{and } \frac{850}{25} = A + B\,(25) - Eq(ii)$$

Solving (i) and (ii), we get

$$25B = \frac{2500}{50} - \frac{850}{25} = \frac{2500 - 1700}{50}$$

$$= \frac{800}{50} = 16$$

$$B = \frac{16}{25}$$

and from (i), $A = 50 - \frac{16}{25} \times 50 = 50 - 32 = 18$

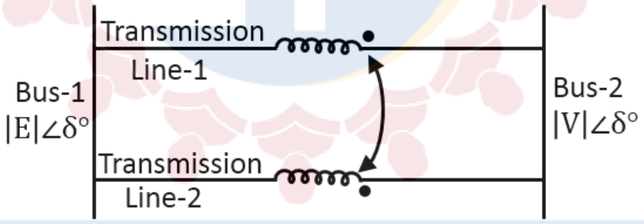
So, at 50 Hz

$$P_h = Af = 18 \times 50 = 900\text{ W}$$

$$P_e = Bf^2 = \left(\frac{16}{25}\right) \times (50)^2 = 1600\text{ W}$$

End of the Solution

44. In the figure shown, self-impedances of the two transmission lines are 1.5j p.u each and $Z_m = 0.5j$ p.u is the mutual impedance. Bus voltages shown in the figure are in p.u. Given that $\delta > 0$, the maximum steady-state real power that can be transferred in p.u from Bus-1 to Bus-2 is



A. $|E||V|$

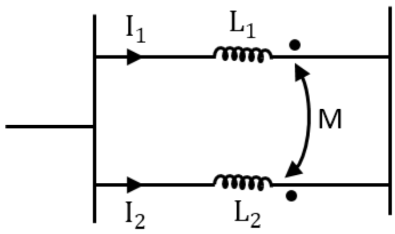
B. $\frac{|E||V|}{2}$

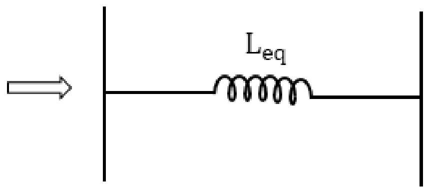
C. $2|E||V|$

D. $\frac{3|E||V|}{2}$

Answer: A

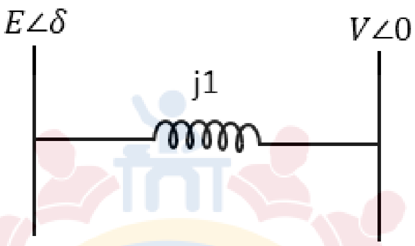
Solution:





$$L_{eq} = \frac{L_1L_2 - M^2}{(L_1 + L_2 - 2M)}$$

$$X_{eq} = \frac{1.5 \times 1.5 - 0.5^2}{1.5 + 1.5 - 2 \times 0.5} = 1 \text{ p.u.}$$

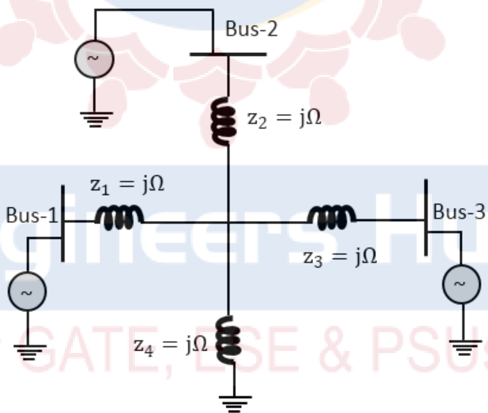


$$P_{max} = \frac{|E||V|}{1}$$

$$P_{max} = |E||V|$$

End of the Solution

45. A 3-Bus network is shown. Consider generators as ideal voltage sources. If rows 1,2 and 3 of the Y_{Bus} matrix correspond to Bus 1,2 and 3, respectively, then Y_{Bus} of the network is



A. $\begin{bmatrix} -4j & j & j \\ j & -4j & j \\ j & j & -4j \end{bmatrix}$

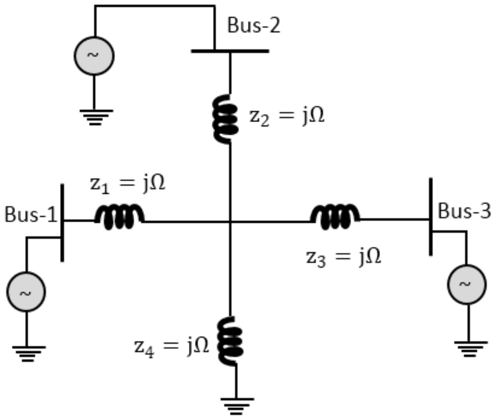
B. $\begin{bmatrix} -4j & 2j & 2j \\ 2j & -4j & 2j \\ 2j & 2j & -4j \end{bmatrix}$

C. $\begin{bmatrix} -\frac{3}{4}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & -\frac{3}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & -\frac{3}{4}j \end{bmatrix}$

D. $\begin{bmatrix} -\frac{1}{2}j & \frac{1}{4}j & \frac{1}{4}j \\ \frac{1}{4}j & -\frac{1}{2}j & \frac{1}{4}j \\ \frac{1}{4}j & \frac{1}{4}j & -\frac{1}{2}j \end{bmatrix}$

Answer: C

Solution:



$$V_1 = j1i_1 + j1(i_2 + i_2 + i_3)$$

$$V_1 = 2ji_1 + ji_2 + ji_3$$

$$V_2 = ji_1 + 2ji_2 + ji_3$$

$$V_3 = ji_1 + ji_2 + 2ji_3$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2j & j & j \\ j & 2j & j \\ j & j & 2j \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} 2j & j & j \\ j & 2j & j \\ j & j & 2j \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} \frac{-3j}{4} & \frac{j}{4} & \frac{j}{4} \\ \frac{j}{4} & \frac{-3j}{4} & \frac{j}{4} \\ \frac{j}{4} & \frac{j}{4} & \frac{-3j}{4} \end{bmatrix}$$

End of the Solution

46. Suppose I_A, I_B and I_C are a set of unbalanced current phasors in a three-phase system. The phase-B zero-sequence current $I_{B0} = 0.1\angle 0°$ p.u. If phase-A current $I_A = 1.1\angle 0°$ p.u and phase-C current $I_C = (1\angle 120^{\circ} + 0.1)$ p.u, then I_B in p.u is

A. $1\angle 240° - 0.1\angle 0°$

B. $1.1\angle 240° - 0.1\angle 0°$

C. $1.1\angle -120° + 0.1\angle 0°$

D. $1\angle -120° + 0.1\angle 0°$

Answer: D

Solution:

$$I_{B0} = \frac{1}{3}(I_A + I_B + I_C)$$

$$0.1 = \frac{1}{3}(1.1\angle 0 + I_B + 1\angle 120° + 0.1)$$

$$I_B = 0.3 - 1.1\angle 0 - 0.1 - 1\angle 120°$$

$$I_B = -0.9 - 1\angle 120°$$

$$I_B = 0.1 + 1\angle 240°$$

$$I_B = 1\angle -120° + 0.1\angle 0°$$

End of the Solution

47. A counter is constructed with three D flip-flops. The input-output pairs are named (D_0, Q_0), (D_1, Q_1), and (D_2, Q_2), where the subscript 0 denotes the least significant bit. The output sequence is desired to be the Gray-code sequence 000, 001, 011, 010, 110, 111, 101, and 100, repeating periodically. Note that the bits are listed in the $Q_2\ Q_1\ Q_0$ format. The combinational logic expression for D_1 is

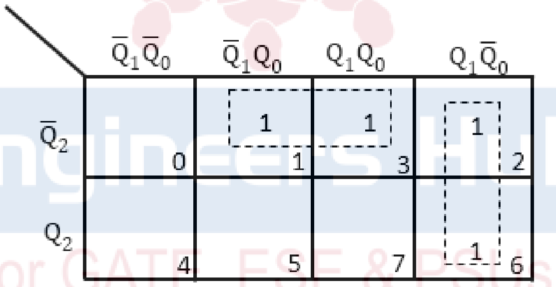
A. $Q_2Q_1Q_0$ B. $Q_2Q_0 + Q_1\overline{Q}_0$

C. $\overline{Q}_2Q_0 + Q_1\overline{Q}_0$ D. $Q_2Q_1 + \overline{Q}_2\overline{Q}_1$

Answer: C

Solution:

Present state $Q_2\ Q_1\ Q_0$	Next state $Q_2^+\ Q_1^+\ Q_0^+$	D_2 D_1 D_0
0 0 0	0 **0** 1	**0**
0 0 1	0 **1** 1	**1**
0 1 1	0 **1** 0	**1**
0 1 0	1 **1** 0	**1**
1 1 0	1 **1** 1	**1**
1 1 1	1 **0** 1	**0**
1 0 1	1 **0** 0	**0**
1 0 0	0 **0** 0	**0**



$$D_1 = \overline{Q}_2Q_0 + Q_1\overline{Q}_0$$

End of the Solution

48. Let A be a 10×10 matrix such that A^5 is a null matrix, and let I be the 10×10 identify matrix. The determinant of $A + I$ is __________

Answer: 1 to 1

Solution:

Given: $A^5 = 0$

$$Ax = \lambda x$$

$$\Rightarrow A^5x = \lambda^5 x\ (\because x \neq 0)$$

$$\Rightarrow \lambda^5 = 0$$

$$\Rightarrow \lambda = 0$$

Eigen values of A + I given $\lambda + 1$

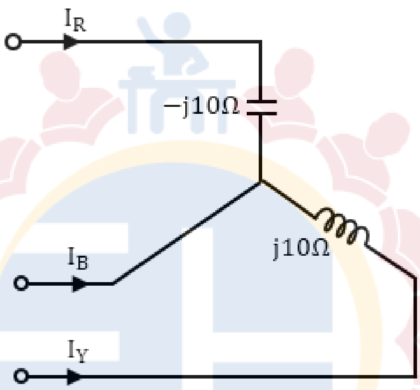
$\because$ Eigen values of $I_A = 1$

Hence $|A + I|$ = Product of eigen values = $1 \times 1 \times 1 \times \ldots 10$ times

$$= 1$$

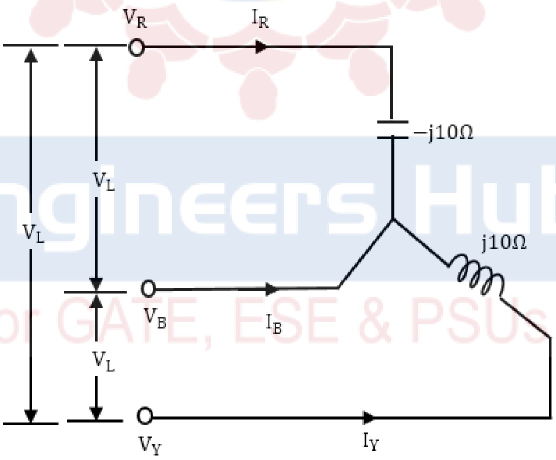
End of the Solution

49. A three-phase balanced voltage is applied to the load shown. The phase sequence is RYB. The ratio $\frac{|I_B|}{|I_R|}$ is ________.



Answer: 1 to 1

Solution:



$$I_R = \frac{V_{RB}}{-j10} = \frac{V_L\angle -60°}{-j10} = \frac{V_L}{10}\angle 30°$$

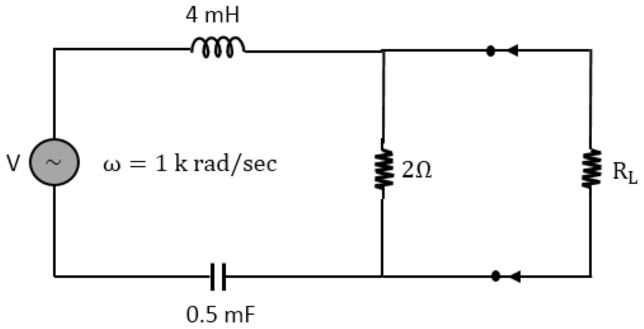
$$I_Y = \frac{V_{YB}}{j10} = \frac{V_L\angle -120°}{-j10} = \frac{V_L}{10}\angle 150°$$

and $I_B = -(I_R + I_Y) = \frac{V_L}{10}\angle -90°$

$$\therefore \left|\frac{I_B}{I_R}\right| = 1$$

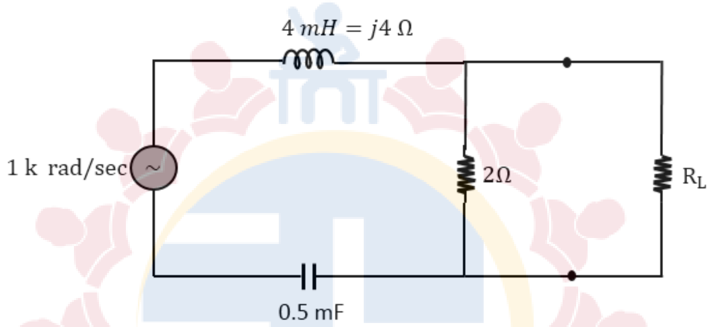
End of the Solution

50. In the given circuit, for maximum power to be delivered to R_L, its value should be ______Ω.
(Round off to 2 decimal places)

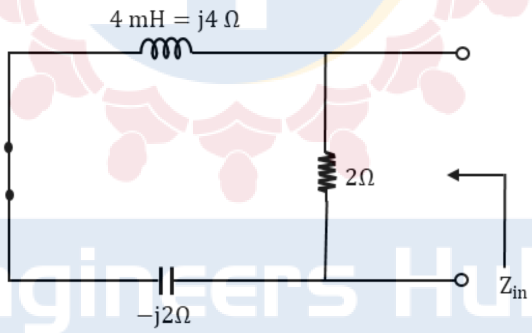


Answer: 1.40 to 1.42

Solution:



$$= \frac{-j}{\omega C} = \frac{-j}{1000 \times 0.5 \times 10^{-3}} = -j2\Omega$$



$$Z_{in} = 2 \| j2 = \frac{j4}{2 + j2} = \frac{j2}{1 + j1}$$

For maximum power transfer,

$$R_L = |Z_{TH}| = \frac{2}{\sqrt{2}} = \sqrt{2} = 1.414\ \Omega$$

End of the Solution

51. One coulomb of point charge moving with a uniform velocity $10\,\hat{x}$ m/s enters the region $x \geq 0$ having a magnetic flux density

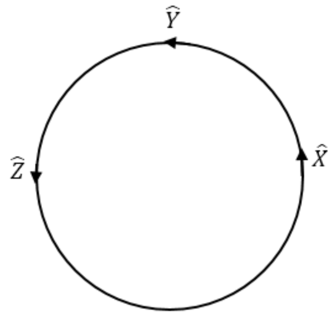
$\vec{B} = (10\,y\hat{x} + 10\,x\hat{y} + 10\hat{z})$ T.

The magnitude of force on the charge at $x = 0^+$ is ___________N.

($\hat{x}, \hat{y}$, and $\hat{z}$ are unit vectors along x-axis, y-axis, and z-axis, respectively)

Answer: 100 to 100

Solution:



Force on a charge moving with $\vec{v}$ velocity due to magnetic field is

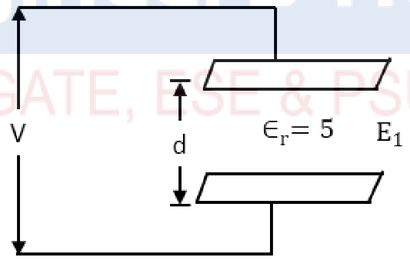
$$\vec{F} = Q(\vec{v} \times \vec{B})$$
$$= 1[10\hat{x} \times (10y\hat{x} + 10x\hat{y} + 10\hat{z})]$$
$$= 10(10x)\hat{z} + 10(10)(-\hat{y})$$
$$= 100\,x\hat{z} - 100\hat{y}$$
$$\vec{F}\big|_{x=0^+} = -100\hat{y}$$
$$|\vec{F}| = \sqrt{(-100)^2} = 100 \text{ Newton}$$

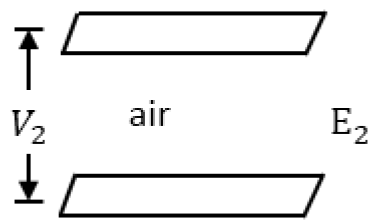
End of the Solution

52. Consider a large parallel plate capacitor. The gap d between the two plates is filled entirely with a dielectric slab of relative permittivity 5. The plates are initially charged to a potential difference of V volts and then disconnected from the source. If the dielectric slab is pulled out completely then the ration of the new electric filed E_2 in the gap to the original electric field E_1 is __________.

Answer: 5 to 5

Solution:





If voltage source is removed then in both cases charge Q is constant.

Case-1: $(Q_1, = Q; V_1 = V)$

$$Q_1 = C_1 V_1$$

$$Q = \frac{\epsilon_0 (5)A}{d} V_1$$

$$Q = 5\left(\frac{\epsilon_0 A}{d}\right) V_1$$

Case-2: $(Q_2 = Q;\ V_2)$

$$Q_2 = C_2 V_2$$

$$Q = \frac{\epsilon_0 (1)A}{d} V_2$$

$$Q = \left(\frac{\epsilon_0 A}{d}\right) V_2$$

Equation (i) is equal to equation (ii)

$$\Rightarrow 5\left(\frac{\epsilon_0 A}{a}\right) V_1 = \left(\frac{\epsilon_0 A}{a}\right) V_2$$

$$\Rightarrow 5V_1 = V_2$$

$$\frac{V_2}{V_1} = 5$$

$$E_1 = \frac{V_1}{d}; E_2 = \frac{V_2}{d}$$

$$\frac{E_2}{E_1} = \frac{V_2 \text{ I d}}{V_1 \text{ I d}}$$

$$\Rightarrow \frac{E_2}{E_1} = \frac{V_2}{V_1}$$

Put equation (iii) in equation (iv)

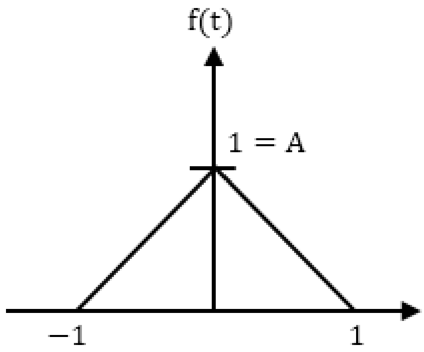
$$\Rightarrow \frac{E_2}{E_1} = 5$$

End of the Solution

53. Consider a continuous-time signal $x(t)$ defined by $x(t) = 0$ for $|t| > 1$, and $x(t) = 1 - |t|$ for $|t| \leq 1$. Let the Fourier transform of $x(t)$ be defined as $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$. The maximum magnitude of $X(\omega)$ is ________.

Answer: 1 to 1

Solution:



Fourier transform, $F(\omega) = A\tau Sa^2\left(\frac{\omega\tau}{2}\right)$

As $A = 1, \tau = 1$

$$F(\omega) = Sa^2\left(\frac{\omega}{2}\right)$$

$$F(\omega)|_{peak} = F(0) = Sa^2(0) = 1$$

∵ Peak value of sampling function occurs at $\omega = 0$.

Peak value = 1

End of the Solution

54. A belt-driven DC shunt generator running at 300 RPM delivers 100 kW to a 200 V DC grid. It continues to run as a motor when the belt breaks, taking 10 kW from the DC grid. The armature resistance is 0.025 Ω, field resistance is 50Ω, and brush drop is 2 V. Ignoring armature reaction, the speed of the motor is ______________ RPM. (Round off to 2 decimal places.)

Answer: 273.00 to 277.00

Solution:

$$I_L = \frac{100 \times 10^3}{200} = 500A$$

$$I_{sh} = \frac{200}{50} = 4\ A$$

$$I_a = 504\ A$$

$$E_q = V + I_aR_a + \text{Brush drop}$$

$$= 200 + 504\ (0.025) + 2V$$

$$= 214.6\ V$$

In motoring case : $VI = 10\ kW, V = 200\ V$

$$I = \frac{10000}{200} = 50A$$

$$I_f = 4A, I_a = I_L - I_f$$

$$= 46\ A$$

$E_b = V - I_aR_a - \text{Brush Drop}$

$$= 200 - 46(0.025) - 2 = 196.85\ V$$

$$\frac{N_m}{N_g} = \frac{E_b}{E_g}$$

$$N_m = \frac{E_b}{E_g} \times N_g$$

$$N_m = \frac{196.85}{214.6} \times 300 = 275.18 \text{ rpm}$$

End of the Solution

55. An 8-pole, 50 Hz, three-phase, slip-ring induction motor has an effective rotor resistance of 0.08 Ω per phase. Its speed at maximum torque is 650 RPM. The additional resistance per phase that must be inserted in the rotor to achieve maximum torque at start is __________Ω (Round off to 2 decimal places). Neglect magnetizing current and stator leakage impedance. Consider equivalent circuit parameters referred to stator.

Answer: 0.50 to 0.54

Solution:

$$N_{max} = 650 \text{ rpm}, P = 8, 50 \text{ Hz}$$

$$S_m = \frac{750 - 650}{750} = 0.1333$$

$$S_m = \frac{R_2}{X_2}$$

$$\therefore\ X_2 = \frac{R_2}{S_m} = \frac{0.08}{0.133} = 0.601\ \Omega$$

$$R_2 = 0.08\ \Omega, X_2 = 0.601\ \Omega$$

Condition for maximum T_{st}

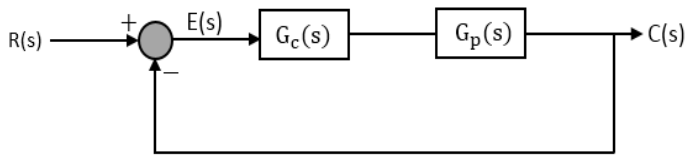
$$\Rightarrow R_2 = X_2$$

$$\therefore R_2 + R_{ext} = X_2$$

$$\therefore R_{ext} = 0.601 - 0.08 = 0.521\ \Omega$$

End of the Solution

56. Consider a closed-loop system as shown. $G_p(s) = \frac{14.4}{s(1+0.1s)}$ is the plant transfer function and $G_c(s) = 1$ is the compensator. For a unit-step input, the output response has damped oscillations. The damped natural frequency is ________ rad/s. (Round off to 2 decimal places.)



Answer: 10.80 to 11.00

Solution:

$$q(S) = S^2 + 10s + 144 = 0$$

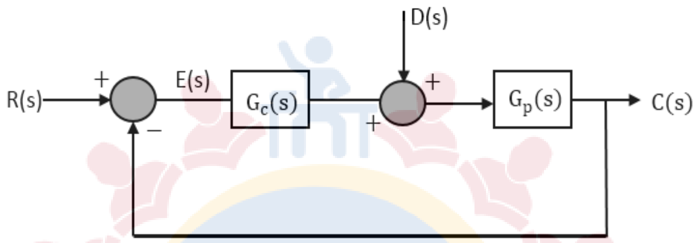
$$\omega_n = 12; \xi = \frac{5}{12}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 12\sqrt{\frac{119}{144}} = 10.90$$

End of the Solution

57. In the given figure, plant $G_p(s) = \frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}$ and compensator $G_c(s) = K\left(\frac{1+T_1s}{1+T_2s}\right)$. The external disturbance input is D(s). It is desired that when the disturbance is a unit step, the steady-state error should not exceed 0.1 unit. The minimum value of K is ________. (Rounded off to 2 decimal places.)



Answer: 9.50 to 9.60

Solution:

$$e_{ss} = \lim_{s\to 0}\left[\frac{sR}{1+G_cG_p} - \frac{sDG_p}{1+G_cG_p}\right]$$

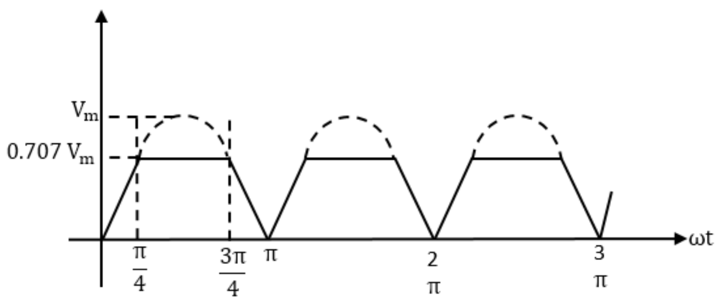
$$R(s) = 0;\ D(s) = \frac{1}{s}$$

$$e_{ss} = \frac{2.2}{1+2.2K} = 0.1$$

$$K_{min} = -10.45$$

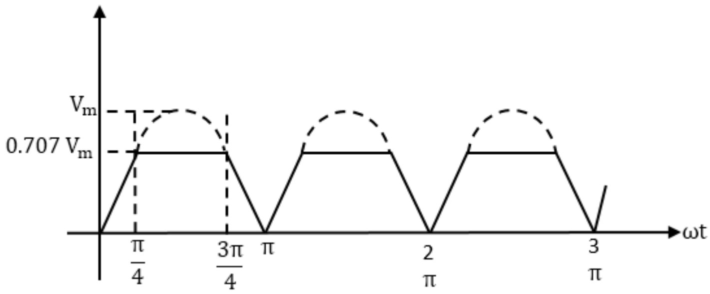
End of the Solution

58. The waveform shown in solid line is obtained by clipping a full-wave rectified sinusoid (shown dashed). The ratio of the RMS value of the full-wave rectified waveform to the RMS value of the clipped waveform is ___________ (Round off to 2 decimal places).



Answer: 1.20 to 1.23

Solution:



We know,

RMS value of full wave rectified sine wave$= \frac{V_m}{\sqrt{2}} = 0.707\ V_m$

For clipped wave from

As $0 \to \frac{\pi}{4}$ and $\frac{3\pi}{4} \to \pi$ wave form are identical

rms value of clipped wave $= \sqrt{\frac{1}{\pi}\left[2\int_0^{\pi/4} V_m^2 \sin^2\omega t + \int_{\pi/4}^{3\pi/4} (0.707V_m)^2\, d\omega t\right]}$

$$= \sqrt{\frac{1}{\pi}\left(\int_0^{\frac{\pi}{4}} \frac{2V_m^2}{2}[1-\cos 2\omega t]d\omega t + \frac{1}{2}V_m^2 \times \frac{\pi}{2}\right)}^{1/2}$$

$$= \left[\frac{1}{\pi}\left[V_m^2\left(\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2}\right) + \frac{V_m^2}{4}\pi\right]\right]^{1/2}$$

$$= \left[\frac{V_m^2}{\pi}\left[\frac{\pi}{4} - \frac{1}{2}\right] + \frac{V_m^2}{4}\right]^{1/2} = 0.5838\ V_m$$

Ratio $= \frac{0.707}{0.5838} = 1.21$

End of the Solution

59. The state space representation of a first-order system is given as

$$\dot{x} = -x + u$$
$$y = x$$

Where, x is the state variable, u is the control input and y is the controlled output. Let $u = -Kx$ be the control law, where K is the controller gain. To place a closed-loop pole at -2, the value of K is ________.

Answer: 1 to 1

Solution:

$$\dot{x} = -x - Kx = x(-kI - I)$$

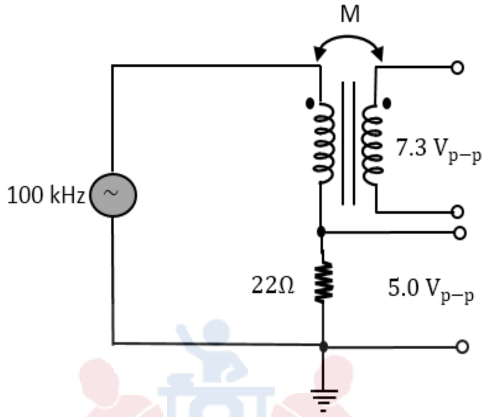
Characteristic equation,

$$|SI + KI + I| = 0$$
$$|(S + 1 + K)I| = 0$$
$$S + 1 + K = 0$$
$$S = -1 - K$$
$$-2 = -1 - K$$

$$K = 1$$

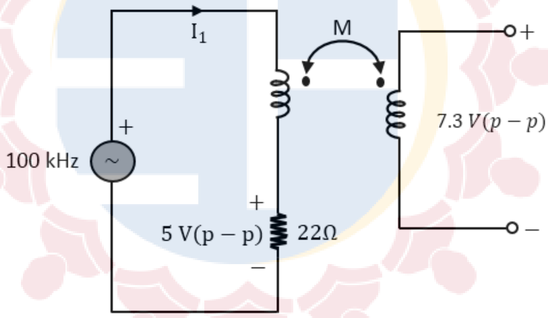
End of the Solution

60. An air-core radio-frequency transformer as shown has a primary winding and a secondary winding. The mutual inductance M between the windings of the transformer is ________μH (Round off to 2 decimal places).



Answer: 50.00 to 52.00

Solution:



$$I_1 = \frac{5}{22}(p - p)$$

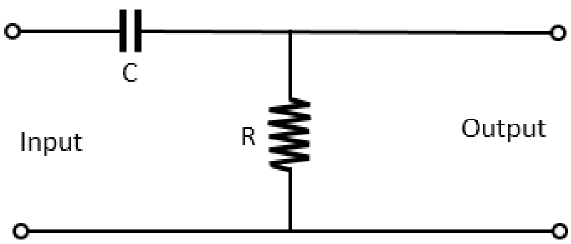
$$V_0 = j\omega MI_1 = 7.3 = (2\pi \times 10 \times 10^3) \times M \times \left(\frac{5}{22}\right)$$

$$M = 51.10\ \mu H$$

End of the Solution

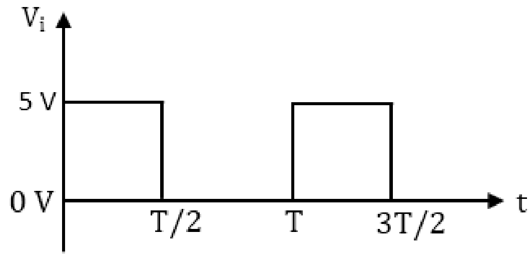
61. A 100 Hz square wave, switching between 0 V and 5 V, is applied to a CR high-pass filter circuit as shown. The output voltage waveform across the resistor is 6.2 V peak-to-peak. If the resistance R is 820 Ω, then the value C is ________, μF.

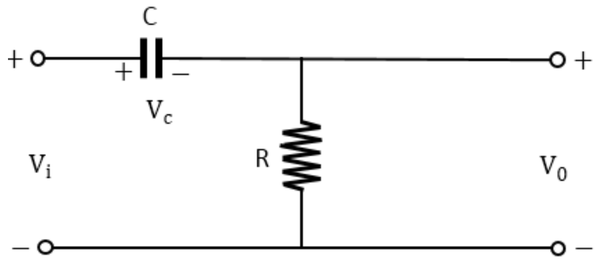
(Round off to 2 decimal places.)

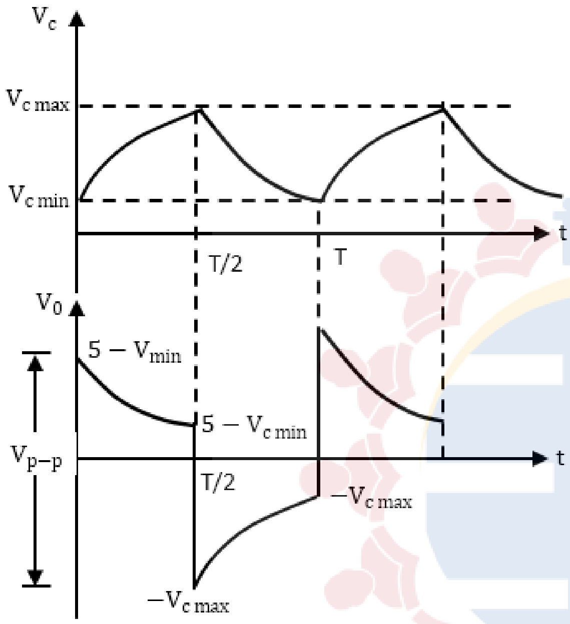


Answer: 12.30 to 12.60

Solution:







$$V_0 = V_i - V_c$$

For 1st half cycle, $V_0 = 5 - V_c$

For 2nd half cycle, $V_0 = -V_c$

$$V_{p-p} = (5 - V_{c\,min}) - (-V_{c\,max})$$

$$6.2 = 5 + V_{c\,max} - V_{c\,min}$$

$$\Rightarrow V_{c\,max} - V_{c\,min} = 1.2 \ldots (\alpha)$$

For first half cycle i.e. $0 < t < \frac{T}{2}$

$$V_c(0^+) = V_c(0) = V_c(0^-) = V_{c\,min}$$

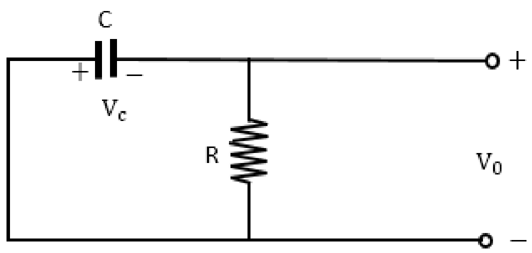
$$V_c(\infty) = 5\text{ V}$$

$$\therefore\ V_c(t) = V_c(\infty) + [V_c(0^+) - V_c(\infty)]e^{-t/\tau}$$

$$V_c(t) = 5 + [V_{c\,min} - 5]e^{-t/2\tau} = V_{c\,max}$$

$$\Rightarrow\ V_{c\,max} = 5 + \left[1 - e^{-T/2\tau}\right] + V_{c\,min}e^{-T/2\tau}$$

For $\frac{T}{2} < t < T$



$$V_c(f) = V_c\left(\frac{T}{2}\right) e^{-t(t-T/2)\tau}$$

$$\therefore V_c(t) = V_{c\,max} e^{-(t-T/2)\tau}$$

$$\text{at } t = T, V_c = V_{c\,min}$$

$$\Rightarrow V_{c\,min} = V_{cmax} e^{-T/2\tau}$$

$$\text{As } V_{c\,max} - V_{c\,max} = 1.2$$

[From (α)]

$$\therefore V_{cmax} - V_{c\,max} e^{\frac{-T}{2t}} = 1.2$$

$$V_{c\,max} = \frac{1.2}{1 - e^{-T/2\tau}}$$

$$\Rightarrow V_{c\,max} = \frac{1.2}{1 - e^{-T/2\tau}} = 5\left[1 - e^{\frac{-T}{2\tau}}\right] + V_{cmin} e^{-2\tau}$$

From (ii)

$$V_{c\,max} = 5[1 - e^{-T/2\tau}] + \left(V_{cmax} e^{\frac{-T}{2\tau}}\right) e^{-T/2\tau}$$

$$V_{c\,max} = 5[1 - e^{-T/2\tau}] + V_{cmax} e^{-T/\tau}$$

$$\Rightarrow V_{cmax}\left[1 - e^{\frac{-T}{\tau}} = 5[1 - e^{-T/2\tau}]\right]$$

$$V_{c\,max} = \frac{5[1 - e^{-T/2\tau}]}{[1 + e^{-T/2\tau}][1 - e^{-T/2\tau}]}$$

Using equation (iii),

$$\frac{1.2}{1 - e^{-T/2\tau}} = \frac{5}{1 + e^{-T/2\tau}}$$

$$\Rightarrow 1.2 + 1.2\, e^{-T/2\tau} = 5 - 5\, e^{-T/2t}$$

$$\Rightarrow 6.2\, e^{-T/2\tau} = 3.8$$

$$e^{-T/2\tau} = \frac{3.8}{6.2} = 0.6129$$

$$\frac{T}{2\tau} = 0.4895$$

as $T = \frac{1}{f} = \frac{1}{100}$ sec

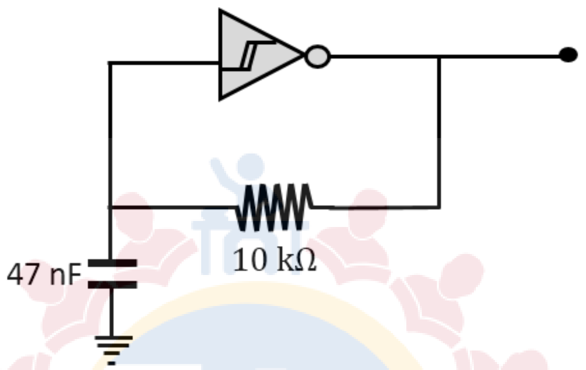
and $\tau = RC = 820\ C$

$$\Rightarrow \frac{1}{(100)(2)(820)C} = 0.4895$$

$$\therefore C = 12.46\ \mu F$$

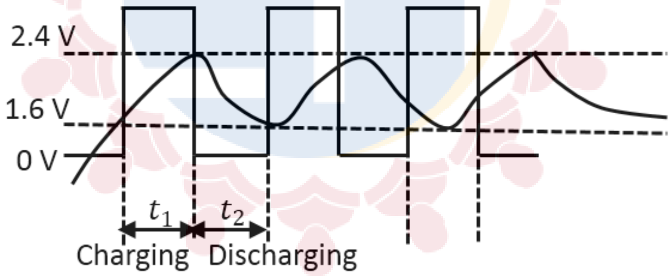
End of the Solution

62. A CMOS Schmitt-trigger inverter has a low output level of 0 V and a high output level of 5 V. It has input thresholds of 1.6 V and 2.4 V. The input capacitance and output resistance of the Schmitt-trigger are negligible. The frequency of the oscillator shown is _________Hz.
(Round off to 2 decimal places.)



Answer: 3150.00 to 3170.00

Solution:



$$V_c(t) = V_{c\,final} + [V_{initial} - V_{c'final}]e^{-t/RC}$$

Charging, $V_c(t) = 5 + (1.6 - 5)e^{t/RC}$

$$= 5 - 3.5\ e^{-t/Rc}$$

$$t = t_1,$$

$$V_c(t) = 2.4\ V$$

$$2.4 = 5 - 3.4\ e^{-t/RC}$$

$$3.4\ e^{-t1/RC} = 2.6$$

$$t_1 = \text{In}\left(\frac{3.4}{2.6}\right) RC = 0.268 \times RC$$

Discharging

$$V_c(t) = 0 + (2.4 - 0)e^{-t/RC}$$

$$= 2.4\ e^{-t/RC}$$

$$t = t_2,$$

$$V_c(t_a) = 1.6\ V$$

$$1.6 = 2.4\ e^{-t2/Rc}$$

$$t_2 = \text{In}\left(\frac{2.4}{1.6}\right) RC = 0.405 \times RC$$

$$T = t_1 + t_2 = (0.268 + 0.405)RC$$

$$T = 0.673\ RC$$

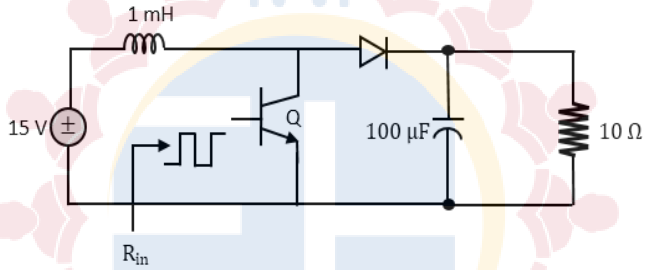
$$f = \frac{1}{0.673RC} = \frac{1}{0.673 \times 10^4 \times 47 \times 10^{-9}}$$

$$f = 3157.46\ \text{Hz}$$

End of the Solution

63. Consider the boost converter shown. Switch Q is operating at 25 kHz with a duty cycle of 0.6. Assume the diode and switch to be ideal. Under steady-state condition, the average resistance R_{in} as seen by the source is _________Ω.

(Round off to 2 decimal places.)



Answer: 1.55 to 1.65

Solution:

Checking for continuous conduction mode

$$\Delta I_L = \frac{\alpha V_s}{fL} = \frac{0.6 \times 15}{25 \times 10^3 \times 1 \times 10^{-3}} = 0.36\ A$$

$$\frac{\Delta I_L}{2} = 0.18\ A$$

$$I_{L\,min} = I_L - \frac{\Delta I_L}{2} = I_S - \frac{\Delta I_L}{2}$$

$$= (9.375 - 0.18) = 9.195 > 0$$

As it is continuous conduction

$$V_0 = \frac{V_S}{1-\alpha} = \frac{15}{1-0.6} = 37.5V$$

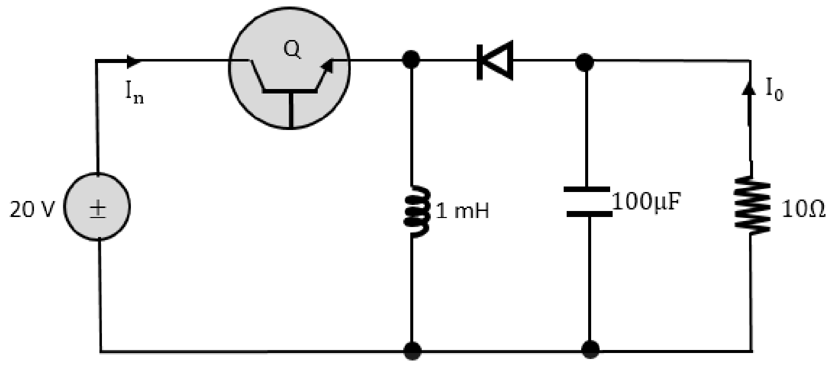
$$I_0 = \frac{V_0}{R} = \frac{37.5}{10} = 3.75V$$

$$\frac{V_0}{V_S} = \frac{I_S}{I_0} = \frac{1}{1-\alpha}$$

$$I_S = \frac{I_0}{1-\alpha} = \frac{3.75}{1-0.6} = 9.375A$$

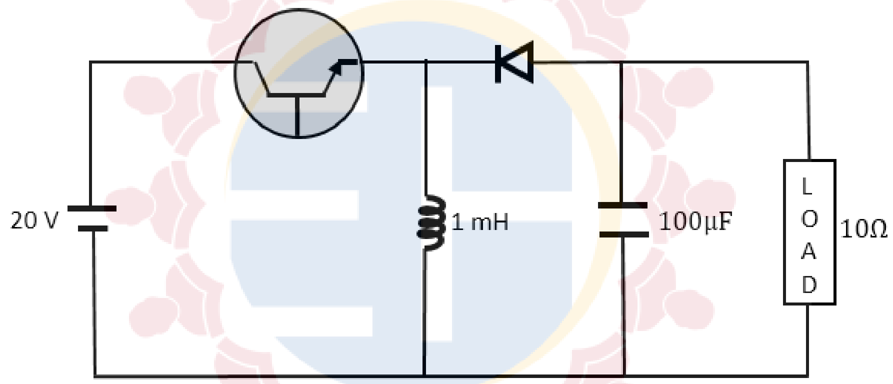
$$R_{in} = \frac{V_s}{I_s} = \frac{15}{9.375} = 1.6\Omega$$

64. Consider the buck-boost converter shown. Switch Q is operating at 25 kHz and 0.75 duty-cycle. Assume diode and switch to be ideal. Under steady state condition, the average current flowing through the inductor is ________A.



Answer: 24 to 24

Solution:



$$\alpha = 0.75, f = 25 \text{ kHz}$$

Assume continuous conduction:

$$V_0 = \frac{\alpha V_s}{1-\alpha} = \frac{0.75 \times 20}{1-0.75}$$

$$V_0 = 60 \text{ V}$$

$$I_0 = \frac{V_0}{R} = \frac{60}{10} = 6\text{A}$$

$$I_L = \frac{I_0}{1-\alpha} = \frac{6}{1-0.75} = 24 \text{ A}$$

$$\Delta I_L = \frac{\alpha V_s}{f_L} = \frac{0.75 \times 60}{25 \times 10^3 \times (1 \times 10^{-3})} = 1.8 \text{ A}$$

$$I_{L\,min} = I_L - \frac{\Delta I_L}{2} = 24 - \frac{1.8}{2} = 24 - 0.9$$

$$(I_{L\,min} = 23.1 \text{ A}) > 0$$

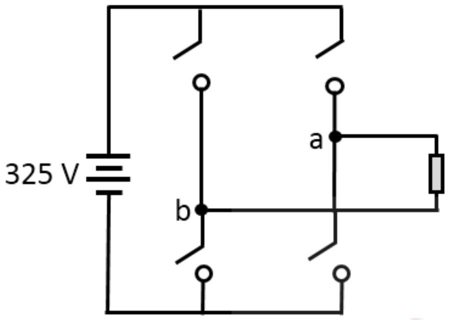
∴ Continuous conduction assumption is correct

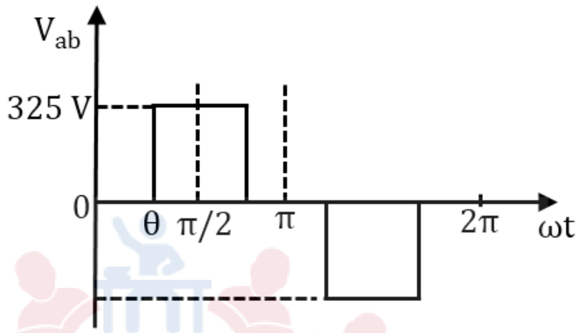
$$I_L = 24 \text{ A}$$

End of the Solution

65. A single-phase full-bridge inverted fed by a 325 V DC produces a symmetric quasi-square waveform across ‘ab’ as shown. To achieve a modulation index of 0.8, the angle θ expressed the degrees should be ________ (Rounded off to 2 decimal places).

(Modulation index is defined as the ratio of the peak of the fundamental component of V_{ab} to the applied DC value.)





Answer: 50.00 to 52.00

Solution:

$$\hat{V}_{01} = m_a V_S = 0.8 \times 325 \times 260 \text{ V}$$

$$\hat{V}_{01} = \frac{4V_S}{\pi} \text{sind} = 260$$

$$\frac{4(325)}{\pi} \text{sind} = 260$$

$$\text{sind} = \frac{260 \times \pi}{4 \times 325} = 0.628$$

$$d = 38.9$$

$$\theta = \frac{\pi}{2} - d = 90^\circ - 38.9 = 51.1$$

End of the Solution